MUMT 618: Computational Acoustic Modeling
Homework #5

1. The system equation for the ideal mass-spring system of Fig. 1 is $\ddot{x} + \omega_0^2 x(t) = 0$, where $\omega_0 = \sqrt{k/m}$. This second-order differential equation has solutions of the form $x(t) = A\cos(\omega_0 t + \phi)$. Derive expressions for A and ϕ in terms of the constants $x(0) = x_0$, $v(0) = v_0$, the displacement and velocity when $t = 0$. [2 pts]

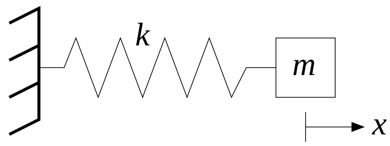

Figure 1: An ideal mass-spring system.

2. Consider longitudinal motion of the three-mass, four-spring system shown in Fig. 2. Write the equations of motion for this system. [1 pt]

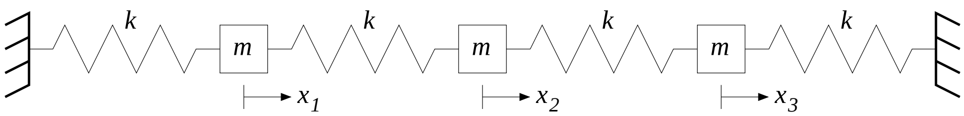

Figure 2: An ideal three-mass, four-spring system: Longitudinal motion.

3. Consider a string extending from $x = 0$ to $x = L$ that is terminated at $x = L$ by the load impedance Z_L. A sinusoidal force disturbance at position x along the string can be described by

$$F(x,t) = \left[C^+ e^{-jkx} + C^- e^{jkx}\right] e^{j\omega t}, \tag{1}$$

where C^+ and C^- are complex traveling-wave amplitudes, $k = \omega/c$ is the wave number, ω is radian frequency, and c is the wave speed. The corresponding velocity is given by

$$V(x,t) = \frac{1}{R}\left[C^+ e^{-jkx} - C^- e^{jkx}\right] e^{j\omega t}, \tag{2}$$

where $R = T/c$ is the real-valued wave impedance of the string with tension T. Derive an expression for the impedance at $x = 0$, or the *input impedance* $Z_{in}(\omega) = F(0,t)/V(0,t)$, in terms of $Z_L, R, \cos(kL)$ and $\sin(kL)$. Then evaluate the result if the string is rigidly fixed at $x = L$. [4 pts]

4. Apply the centered finite difference scheme to the series mass-spring-damper system discussed in class (provide an explicit solution for arbitrary x_0 and v_0 and show the derivation of the equations). Then implement the equations in Matlab and compare the results with those of the backward finite difference (include the backward finite difference solution in your script and then plot the responses together). [5 pts]