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Physics-based modeling of the yehu, a Chinese bowed string instrument

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Abstract:	The yehu is a traditional Chinese bowed string instrument featuring a resonator carved from a coconut shell, a shell-based bridge, and two silk strings. This study combines a physics-based bowed string model with acoustic measurements to enable realistic sound synthesis of the yehu. The proposed physical model comprises a linear stiff string with frequency-dependent losses driven by a moving bow, with the excitation force computed through a nonlinear friction curve linking bow force and bow velocity. A finite difference scheme is employed to simulate transverse string vibrations in the time domain. Both the bridge admittance and radiation responses are obtained from measurements.
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1. INTRODUCTION

The yehu is a Chinese bowed string instrument that features two fixed silk strings (typically tuned to F4 and C5), with the bow positioned between them, as shown in the left panel of Fig. 1. The instrument does not have a fingerboard; instead, players use their fingers to stop the strings directly to vary the pitch. The two strings are in close proximity, and the performer typically presses both strings simultaneously with a single finger. At the bridge, the strings press against a seashell bridge mounted on a wooden plate, which is rigidly attached to the carved coconut resonator. The effective vibrating length of the yehu string extends from the restraining loop to the bridge. However, to capture the string-bridge coupling dynamics, the modeled strings are defined over the extended domain from the restraining loop to the stub, as shown in the right panel of Fig. 1.

Bowed string instrument modeling requires an appropriate description of the friction between the bow and the string. Static friction models describe the friction force as a function of relative velocity, such as the friction curve model,¹ while dynamic models include an elasto-plastic formulation² and a thermal friction model.³ Both types have been extended to time-domain simulations, using digital waveguide⁴ and finite-difference schemes.⁵ Beyond friction, the vibration of the string is shaped by its coupling with the bridge and the instrument body. Several approaches have been proposed to model this interaction. A wave-propagation model can simulate coupling using both reflection and transmission filters,⁶ modal approaches introduce coupling via projection onto the modal basis, resulting in globally coupled modes.⁷ Both approaches allow string-bridge coupling and simulation of wolf notes. The instrument body and its radiation are commonly modeled as a linear time-invariant (LTI) system, enabling the use of digital filter design techniques or direct convolution with measured impulse responses to reproduce resonant and radiative characteristics. The body response can be implemented either via convolution or through infinite impulse response (IIR) filter design. Compared to direct-form realizations, parallel second-order IIR filter structures generally exhibit improved numerical robustness.⁸ Furthermore, IIR design methods based on logarithmic frequency resolution can significantly reduce the required filter order.⁹

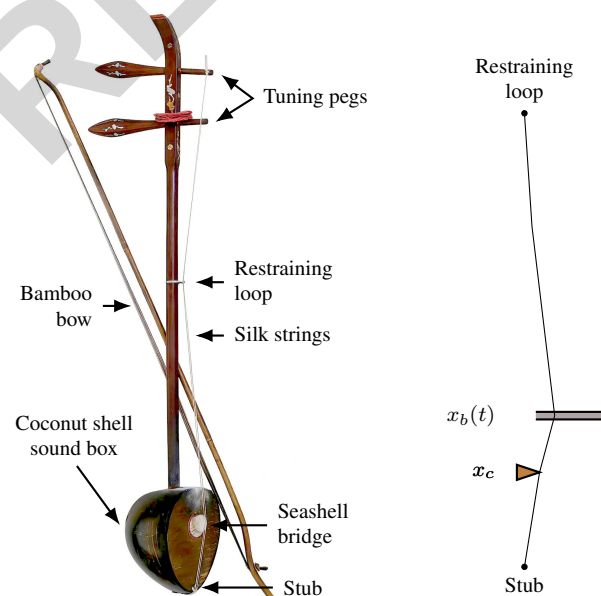


Figure 1: Left: A picture of the author's yehu with parts labeled. The bow hair passes between the two strings. Right: Side view of a simplified model of the yehu. The string is simply supported at the restraining loop and stub. The string is coupled to the bridge and body at x_c . The bow interact with string at $x_b(t)$, respectively.

2. MODEL DESCRIPTION

A. STRING EQUATIONS

The proposed yehu model can be divided into two main force components : bridge-string coupling force and bowing excitation force. The partial differential equation governing the time evolution of the string's transversal displacement $u(x, t)$ can be written as:

$$(\rho A \partial_{tt} - T \partial_{xx} + EI \partial_{xxxx} + 2\rho A (\sigma_0 \partial_t - \sigma_1 \partial_{txx})) u = -J_f f_f + J_c f_c. \quad (1)$$

Here, $x \in \mathcal{D}_L = \{x \in \mathbb{R} \mid 0 \leq x \leq L\}$ and L is the string length (m). f_f and f_c are the friction force and bridge coupling force, respectively, acting on the string. J_f and J_c are spatial distribution functions that map the forces from the interaction point to the string domain. ρ , A , T , E , and I denote mass density ($\text{kg} \cdot \text{m}^{-3}$), cross-sectional area (m^2), tension (N), Young's modulus (Pa), and area moment of inertia (m^4), respectively. The parameters σ_0 (s^{-1}) and σ_1 ($\text{m}^2 \cdot \text{s}^{-1}$) are loss coefficients accounting for frequency-independent and frequency-dependent losses. The boundaries of each string are simply supported:

$$u(0, t) = u(L, t) = 0, \quad \partial_x^2 u(0, t) = \partial_x^2 u(L, t) = 0. \quad (2)$$

B. STRING-BRIDGE COUPLING

The bridge is modeled as a mass-spring-damper (MSD) system representing a modal expansion of a measured bridge admittance. The result is a set of M ordinary differential equations,

$$M_m \frac{d^2 q_m}{dt^2} + R_m \frac{dq_m}{dt} + K_m q_m = -f_c, \quad (3)$$

where q_m is the modal displacement (m), M_m is the modal mass (kg), K_m is the modal spring constant (N/m), and R_m is the modal damping term (kg/s), which are defined by

$$R_m = M_m \sigma_m \omega_m, \quad K_m = M_m \omega_m^2, \quad (4)$$

with σ_m the modal damping ratio, and ω_m the resonant frequency in rad/s of each mode. In the instrument model considered here, the strings are tightly coupled to the bridge and exhibit negligible collision. Therefore, the displacement of the strings at the bridge position may be assumed to be equal to the bridge displacement, as follows:

$$\int_{\mathcal{D}_L} J_c(x_c) u(x, t) dx = \sum_{m=1}^M q_m(t). \quad (5)$$

C. BOW EXCITATION

The bow-string interaction is modeled using a classical friction curve proposed by J.H. Smith and J. Woodhouse.¹ In this model, the relative velocity between the string and the bow is defined as

$$v_r(t) = \int_{\mathcal{D}_L} J_b(x_b) \frac{\partial u(x, t)}{\partial t} dx - v_b(t), \quad (6)$$

where $v_b(t)$ is the bow velocity, and $J_b(x_b)$ is a spatial distribution function that accounts for the finite width of the bow hair centered at position x_b . During the slip, the friction force f_f is modeled as

$$f_f(t) = F_N \phi(v_r), \quad (7)$$

where F_N is the bow force and the dimensionless friction coefficient is given by¹

$$\phi(v_r) = \text{sign}(v_r) \left(0.4e^{-\frac{|v_r|}{0.01}} + 0.45e^{-\frac{|v_r|}{0.1}} + 0.35 \right). \quad (8)$$

During the stick, the string adheres to the bow, and the relative velocity satisfies the constraint $v_r(t) = 0$. In this case, the static friction force acts as a constraint to ensure:

$$\int_{\mathcal{D}_L} J_b(x_b) \frac{\partial u(x, t)}{\partial t} dx = v_b(t). \quad (9)$$

The transition between stick and slip is governed by a threshold condition on the friction force. Specifically, the stick is maintained as long as

$$|f_f(t)| \leq F_s, \quad (10)$$

where $F_s = \mu_s F_N$ denotes the maximum static friction force. μ_s is usually set to a constant value of 1.2. When this bound is exceeded, the system transitions to the slip regime, where the friction force is determined by Eqs. (7-8) above.

D. ENERGY ANALYSIS

Taking the inner product of Eq. (1) with $\partial_t u$ over $\mathcal{D}_L$, multiplying Eq. (3) with $\partial_t q_m$, and summing the modes together, the energy-balance of the system can be derived:

$$\frac{d\mathcal{H}}{dt} = \mathcal{P} - \mathcal{Q} + \mathcal{B}_s|_0^L. \quad (11)$$

Under simply supported boundary conditions, the boundary term $\mathcal{B}_s$ vanishes. Here, the string energy $\mathcal{H}_s$, bridge energy $\mathcal{H}_c$ are given by

$$\mathcal{H}_s = \frac{\rho A}{2} \|\partial_t u\|_{\mathcal{D}_L}^2 + \frac{T}{2} \|\partial_x u\|_{\mathcal{D}_L}^2 + \frac{EI}{2} \|\partial_{xx} u\|_{\mathcal{D}_L}^2, \quad (12a)$$

$$\mathcal{H}_c = \sum_{m=1}^M \left[\frac{M_m}{2} \left(\frac{dq_m}{dt} \right)^2 + \frac{K_m}{2} q_m^2 \right], \quad (12b)$$

$$\mathcal{H} = \mathcal{H}_s + \mathcal{H}_c. \quad (12c)$$

The power supplied to the system, $\mathcal{P}$, is given by

$$\mathcal{P} = f_f \int_{\mathcal{D}_L} J_b(x_b) (\partial_t u) dx. \quad (13)$$

Power dissipation related to the strings $\mathcal{Q}_s$ and the bridge $\mathcal{Q}_c$ are given by

$$\mathcal{Q}_s = 2\rho A (\sigma_0 \|\partial_t u\|_{\mathcal{D}_L}^2 + \sigma_1 \|\partial_{tx} u\|_{\mathcal{D}_L}^2), \quad (14a)$$

$$\mathcal{Q}_c = \sum_{m=1}^M R_m \left(\frac{dq_m}{dt} \right)^2, \quad (14b)$$

$$\mathcal{Q} = \mathcal{Q}_s + \mathcal{Q}_c. \quad (14c)$$

3. PHYSICAL CHARACTERIZATION

A. STRING PARAMETERS

The strings of the yehu are traditional woven silk strings and their physical parameters are determined through a series of measurements. The string density was determined by measuring the mass and the volume of multiple string samples. The mass of each sample was measured using a high-precision balance and the volume of each sample was estimated by assuming that each sample is a perfect cylinder and using a caliper to measure the radius of each string. The Young's modulus, tension, and damping coefficients were identified via an inverse modeling procedure¹⁰ from force signal measurements of a silk string excited via the wire-breaking method. To isolate the string from body resonances, the measurement was conducted on an anti-vibration optical table using custom 3D-printed sensor bridges, as shown in the left panel of Fig. 2. These measurements form the basis for the parameterization of the string model used in the subsequent simulations and these parameters are listed in Table 1.

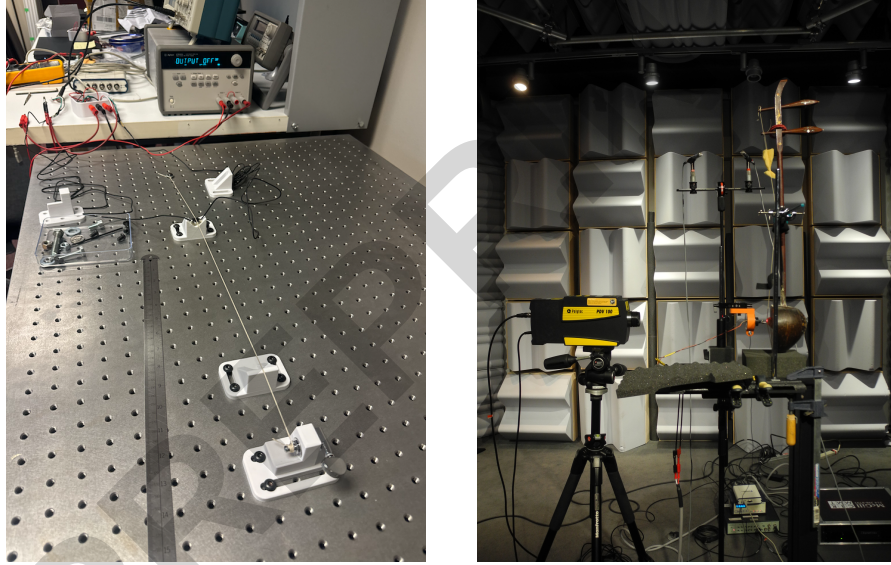


Figure 2: Left : Wire-breaking method setup. Right : Measurement setup in a hemi-anechoic chamber.

	String 1 (F4)	String 2 (C5)	
L	0.38	0.38	m
ρ	1250	1350	kg/m ³
r	0.55	0.40	mm
T	50.61	64.29	N
E	1×10^{10}	9.5×10^9	Pa
σ_0	0.9754	1.5884	s ⁻¹
σ_1	0.0026	0.0032	m ² /s

Table 1: Measured and derived silk string parameters.

B. BRIDGE ADMITTANCE

The bridge admittance was obtained by exciting the bridge using an impact hammer (PCB Model 086C01), and capturing the resulting velocity response with a Laser Doppler vibrometer (Polytec PDV-100). The measurement setup is illustrated in the right panel of Fig. 2. In Fig. 2, the orange device is an actuator that drives the impact hammer,¹¹ while the yellow device is the vibrometer. The blue curve on the left panel of Fig. 3 displays the measured bridge admittance, while the blue curve on the right panel displays the measured bridge velocity in response to a hammer impact.

The measured admittance is then processed using the filter diagonalization method¹² to extract the modal parameters. With the identified modal parameters, the mass terms, spring constant terms and damping terms in Eq. (3) can be derived. A finite difference scheme is designed to simulate the bridge motion in the time domain. The red dashed curves in Fig. 3 show the simulation results of the MSD system with 11 modes together with the measured hammer impact excitation.

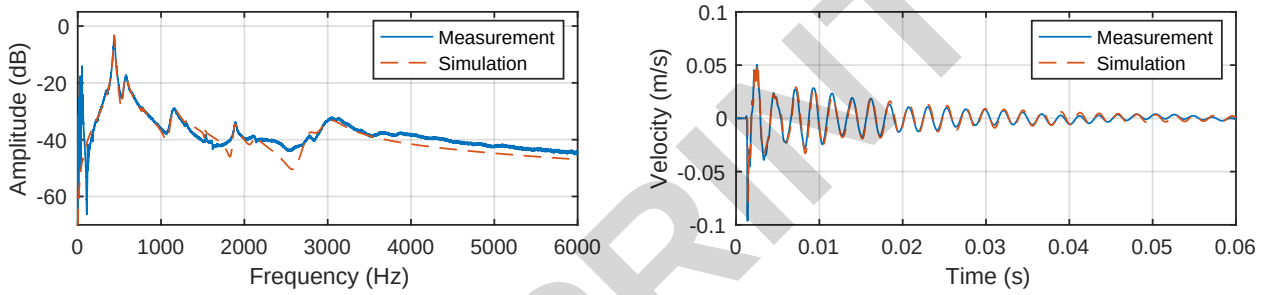


Figure 3: Measured and simulated bridge admittance and response.

C. BODY RADIATION

The transfer function from bridge force to radiated sound pressure was obtained from impact hammer measurements conducted in a hemi-anechoic chamber with a pair of microphones (HBK Type 4190). A single pair of microphones was used to measure at two different positions. One configuration placed the microphones 30 cm behind the soundbox, hereafter referred to as the recording position. Another configuration placed them near the tuning pegs (see the right panel of Fig. 2), approximating the player's ear position during performance, hereafter referred to as the player position. To improve reliability and reduce measurement uncertainty, five repeated measurements were conducted under identical conditions. Furthermore, an averaging operation of five measurements was performed and a high-pass filtering process with a cutoff frequency of 100 Hz was applied to minimize the error and suppress low-frequency noise components. For simplicity and compact presentation, mono signals were obtained by arithmetic averaging of the binaural recordings and used in the following filter design. In practical sound synthesis applications, however, maintaining the full binaural signals at each recording position and designing corresponding channel-specific filters is preferable. The processed results are shown by the blue curve in Fig. 5, with the upper panel corresponding to the recording position and the lower panel to the player position.

4. SIMULATION AND SYNTHESIS

A. FINITE DIFFERENCE IMPLEMENTATION

An energy-balanced finite difference scheme is applied to numerically solve the continuous domain equations in Section 2. String displacement $u(x, t)$ is discretized into grid functions, defined at positions $x = lh$ and times $t = nk, n \in \mathbb{N}$. h is the grid spacing (in m) and $l \in [0, \dots, N]$. $k = 1/F_s$ is the time step,

in s , with F_s the sample rate in Hz. For a continuous function $u(x, t)$ defined by $x \in \mathcal{D}_{\mathcal{L}}$ and $t \in \mathbb{R}^+$, u_l^n is a grid function approximation of $u(lh, nk)$. For a given time step k , the system is stable under the condition, $h \geq h_{min}$, where

$$h_{min}^2 = \frac{Tk^2}{2\rho A} + \frac{k\sigma_1}{2} + \frac{k}{2} \sqrt{\left(\frac{Tk}{\rho A} + 4\sigma_1\right)^2 + \frac{16EI}{\rho A}}. \quad (15)$$

In the finite difference scheme, the string update requires the computation of both the coupling and friction force terms at the current time step, whereas the bridge update requires only the computation of the force term. The coupling force term requires a coupling matrix to solve the coupling force under the assumption of rigid string-bridge coupling. With the computed coupling force, 11 mass-spring-damper elements, corresponding to the above admittance simulation in Section 3.B, can be updated independently. The friction force term is computed by solving the nonlinear equation Eq. (8) at each time step using the Newton–Raphson iteration. A discretized energy check is implemented on each time step based on the continuous energy analysis in Section 2.D.

B. STEADY STATE MOTION

Figure 4 shows the simulated bridge force, bridge velocity, string relative velocity, and system energy error over a representative time interval in the steady-state regime. The simulation sample rate is set to 44.1 kHz, and the string parameters correspond to those of the F string listed in Table 1. The bow force and bow velocity are set to constant values of 0.2 N and 0.2 m/s, respectively. The bridge is coupled to the string at a position 30 cm from the restraining loop, while the bowing position is located 25 cm from the restraining loop. A third-order Lagrange interpolation is employed to evaluate the bow force at off-grid positions. The tolerance of the Newton–Raphson algorithm is set to 1×10^{-12} , with a maximum of 100 iterations. However, in this setup, the number of iterations required never exceeded 9, indicating good convergence behavior. From Fig. 4, the bridge force exhibits a sawtooth-like waveform, while the bridge velocity remains relatively smooth. The string relative velocity demonstrates the stick–slip behavior: intervals of near-zero relative velocity (sticking phase) are followed by sharp negative peaks associated with rapid slipping events. The system energy error remains on the order of machine precision during the whole simulation period, confirming the numerical stability of the scheme.

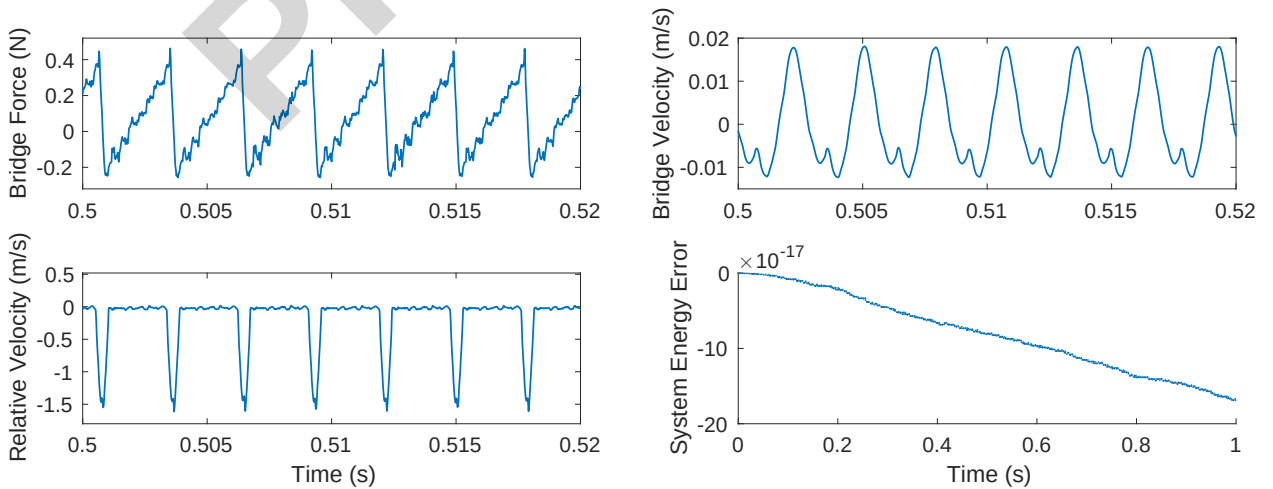


Figure 4: Simulated steady state waveform and system energy error.

5. BODY RADIATION FILTER

The transfer function from bridge force to microphone sound pressure is approximated using a parallel bank of second-order infinite impulse response (IIR) filters. The red dashed curves in Fig. 5 show the results of fitting 25 second-order sections to the measured data with warped poles processing.⁹ The upper panel corresponds to the measurement at the recording position, while the lower panel shows the result at the player position. As observed in Fig. 5, the fitted responses are in good agreement with the measured data, indicating an reasonable approximation.

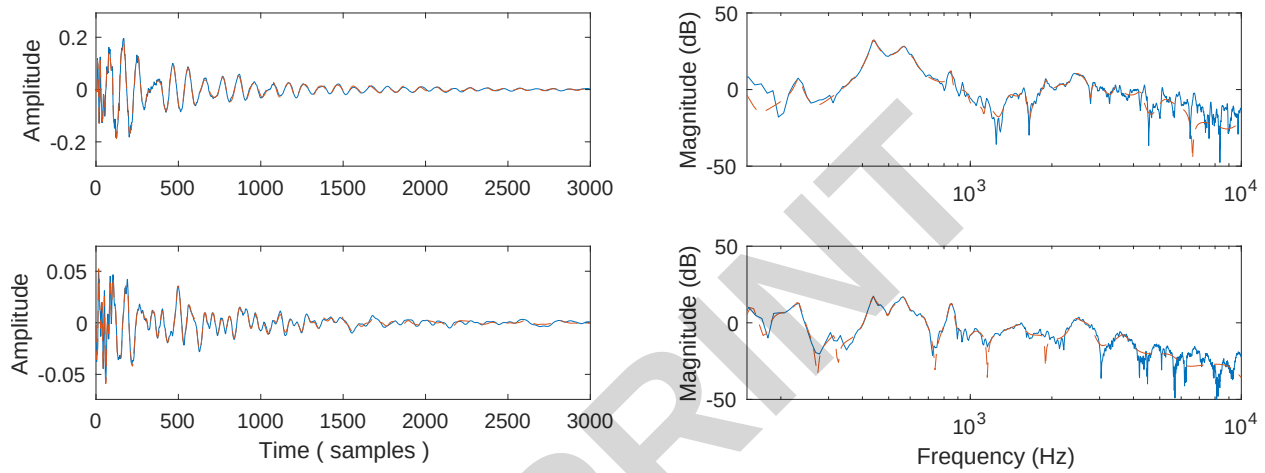


Figure 5: Blue curve : processed measurement results. Red dashed curve : parallel second-order IIR filter fitting. Upper panel : recording position. Lower panel : player position.

The synthesized output is subsequently obtained by filtering the simulated bridge force. The resulting steady-state waveforms, shown using normalized amplitude, and their corresponding spectra are presented in Fig. 6. The upper-left panel corresponds to the recording position, while the lower-left panel corresponds to the player position; the right panel shows the corresponding frequency-domain results. The spectrum of the recording position exhibits a higher overall magnitude, primarily due to the shorter direct distance between the microphone and the soundbox. However, in the 1–1.8 kHz range, the player-position spectrum exhibits a comparable energy level to that of the recording position. This spectral contrast largely accounts for the perceived timbral differences between listening positions during real performance. Sound samples and simulation codes are available on the companion website.¹

6. CONCLUSION

This paper presents a physics-based model of the yehu, implemented using a finite-difference scheme with measurement-informed parameterization. The simulations reproduce the characteristic stick–slip behavior of bowed string instruments and capture the bridge dynamics under rigid string–bridge coupling. A parallel IIR filter bank is employed to model the body and radiation, enabling sound synthesis with perceptual characteristics consistent with real instrument recordings. Future work will focus on model improvement and playability analysis. Extensions include incorporating finger–string contact interactions, coupling both two strings with the bridge, and exploring alternative friction formulations. Further investigation of parameter design and gesture control will be conducted to evaluate and improve the playability of the model.

¹Companion website with sound samples and simulation codes: <https://zzhenjason.github.io/isma26yehu/>

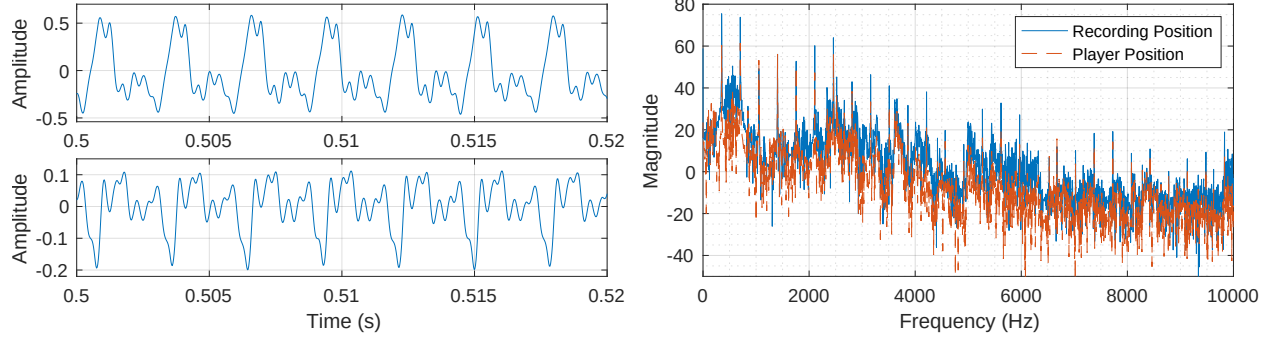


Figure 6: Filtered signal obtained by inputting the simulated bridge force (shown in Fig. 4) to the parallel second-order IIR filter banks. Upper-left panel: waveform of the recording position. Lower-left panel: waveform of the player position. Right panel : spectrums of two positions.

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