

OCTOBER 24 2025

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Proc. Mtgs. Acoust. 58, 035019 (2025)
<https://doi.org/10.1121/2.0002165>



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A study of impedance of brass instruments and mouthpieces

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The input impedance of a brass instrument or a mouthpiece can be analyzed to infer some aspects of playability, and can be calculated from the instrument or mouthpiece geometry. However, because the geometry is not always precisely known or measurable, the impedance is not always straightforward to calculate. In order to work toward a method for calculating the impedance of a full brass instrument from only its geometry, it is necessary to isolate each component of the instrument. A study has been performed to compare the impedance calculated with the transfer matrix and finite element methods to the measured impedances of mouthpieces and the full instrument, with a focus on the trumpet. In addition, the geometric parameters of a trumpet mouthpiece have been estimated from the measured impedance.

1. INTRODUCTION

The acoustic behaviour of a brass instrument can be determined in part from its acoustic impedance, in conjunction with playing simulations and playability studies. A calculation of the impedance of an instrument from its geometry presents several challenges, including accurately determining the geometry of the instrument, particularly its mouthpiece and bell, and understanding how small uncertainties in the geometry, particularly of the mouthpiece, can affect the impedance calculation for the overall instrument.

To this end, a detailed study involving parametrization of the mouthpiece geometry and comparison of the calculated impedance with the measured impedance, has been performed. Aspects of playability, such as harmonicity, are compared for several calculation methods. In addition, the effect of varying individual mouthpiece geometry parameters on the impedance curves of both mouthpiece and instrument is presented.

A study involving variations in the cup shape of trumpet mouthpieces has previously performed with the use of professional players and a perceptual study, as well as simulations and spectral analysis (Zicari et al., 2013). A study involving the variation of the cup depth of a physical mouthpiece was reported by Poirson et al. (2005), who determined that there is a perceptible timbre difference when the tone is produced by an artificial mouth or with a simulation. It was also determined that players tend to unconsciously vary their technique to compensate when presented with different mouthpieces and cannot play consistently enough to produce tones with distinguishable timbres, which is why simulations or artificial blowers are favoured in such studies. An investigation involving several (French) horn mouthpieces (Plitnik and Lawson, 1999) determined the acoustic parameters (such as the Q values of impedance peaks) that correlated most with the preferences of players.

An extension of the technique of calculating impedance from geometry is to estimate the geometry of an instrument or an instrument component from a measured impedance. A similar calculation has previously been done for the bell of a trombone and that of a clarinet (Hélie et al., 2014). In this case, we estimate the geometry of a trumpet mouthpiece from an impedance measurement.

The purpose of this work comprises the following:

1. To use the transfer matrix method (TMM) to fit the parametrized geometry of a mouthpiece by means of the measured impedance, when either the linear impedance or the logarithmic impedance (in dB) is used for the fit.
2. To determine which of three excitation methods is most appropriate in a finite element method (FEM) simulation for estimating the impedance is a system where plane wave propagation is assumed.
3. To compare a TMM calculation including Bessel functions with that involving the “wide pipe” approximation (see §2.A), and also to compare TMM calculations starting from the measured geometry and a parametrization thereof.
4. To determine which calculation method, if any, best reproduces an aspect of the impedance curve related to playability, specifically the inharmonicity of the impedance curve. This includes calculations starting from the fitted geometries in 1. above, which are not calculation methods per se, but it is nevertheless interesting to compare these results, as well as to check the acoustic properties of the fitted geometries.

The models and calculation techniques used for the impedance calculation are described in §2. The studies associated with evaluating the various calculation techniques found in this paper are described in §3. The results of the work are given in §4, and the conclusions can be found in §5.

2. IMPEDANCE CALCULATION METHODS

A. TRANSFER MATRIX METHOD

The transfer matrix method (TMM) allows for the calculation of the input impedance of an axially symmetric segment of an air column, based on the impedance at the outlet, given knowledge of the geometry of the segment. The calculation is as follows:

$$\begin{bmatrix} P_0 \\ U_0 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} P_L \\ U_L \end{bmatrix} \quad (1)$$

where P_0 and U_0 are the pressure and volume velocity at the inlet of each segment, P_L and U_L are those at the outlet, corresponding to the load, and a , b , c , and d are the elements of the transfer matrix for that particular segment. The segment could be a single cone or cylinder, or any other contiguous shape, such as a mouthpiece, for example.

The impedance itself is the ratio of the acoustic pressure at a given location to the acoustic volume velocity at the same location. Therefore, Eq. 1 can also be written as

$$Z_{\text{in}} = \frac{b + aZ_L}{d + cZ_L} \quad (2)$$

where each parameter in the equation is a function of frequency and complex, and viscothermal losses near the walls are taken into account.

For the calculations of the viscothermal losses, we have used either the “wide-pipe” approximation (Keefe, 1984) or the expressions involving Bessel functions (Chaigne and Kergomard, 2016), as specified in the relevant sections. Unless otherwise specified, we use the “wide-pipe” approximation, as it is a much faster calculation and assumed to produce results very close to the calculation involving Bessel functions. This assumption is tested with a direct comparison of the two calculations on the measured mouthpiece geometry (see §3.C).

B. FINITE ELEMENT METHOD

The finite element method (FEM) involves calculations from physical principles of the acoustic pressure and acoustic volume velocity, as well as other quantities, in small segments of volume inside an air column. The calculations are performed using an axisymmetric geometry in COMSOL, based on the same geometry used for the TMM calculation. The mesh used for the calculation consists of triangular elements with dimensions between 0.001 and 0.5 mm. The wall admittance (Chaigne and Kergomard, 2016; Cremer, 1948) is defined to model the viscothermal losses.

The excitation at the inlet is provided by means of three different built-in components, in order that they may be compared. The first is a pressure impulse of 1 Pa at the plane of the inlet. The second is a velocity of 1 m/s at the plane of the inlet, directed normally into the mouthpiece (in the direction of the axis of the mouthpiece). The third is a normally directed acceleration of 1 m/s² at the inlet. The input impedance is calculated as the ratio of the acoustic pressure to the acoustic velocity at the center point of the inlet. Theoretically, in a system that behaves in a linear manner with plane wave propagation, the impedance should not depend on which excitation method is used, and the result should closely match the TMM result. This has been tested and is true for a simple cylindrical air column closed at the outlet. However, it is important to understand the result of varying the excitation method on a more complex system that is pertinent to the study of musical instruments, such as a mouthpiece.

i. Estimating transfer matrix elements from a FEM simulation

To calculate the transfer matrix elements from a FEM simulation, the “two-load” calculation described in Lefebvre (2010) is used. This involves two different end conditions; for example, a closed and an open end, with corresponding values of pressure and volume velocity at the outlet, $P_{\text{out}1}$, $U_{\text{out}1}$, $P_{\text{out}2}$, and $U_{\text{out}2}$. The matrix elements a , b , c , and d are given by the solution to Eq. 3.

$$\begin{bmatrix} P_{\text{out}1} & Z_0 U_{\text{out}1} & 0 & 0 \\ 0 & 0 & P_{\text{out}1} & Z_0 U_{\text{out}1} \\ P_{\text{out}2} & Z_0 U_{\text{out}2} & 0 & 0 \\ 0 & 0 & P_{\text{out}2} & Z_0 U_{\text{out}2} \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} P_{\text{in}1} \\ Z_0 U_{\text{in}1} \\ P_{\text{in}2} \\ Z_0 U_{\text{in}2} \end{bmatrix} \quad (3)$$

where $P_{\text{in}1}$, $U_{\text{in}1}$, $P_{\text{in}2}$ and $U_{\text{in}2}$ are the pressure and volume velocity at the inlet of each segment corresponding to the first and second load conditions, and Z_0 is the characteristic impedance at the inlet.

C. CALCULATION OF THE WHOLE INSTRUMENT INPUT IMPEDANCE

To calculate the instrument impedance from the mouthpiece geometry, the transfer matrix elements are calculated using TMM (see Eq. 2), and the measured impedance corresponding to that of the instrument without mouthpiece is used, as described in §3.A of Jackson and Scavone (2024). The result is the equivalent of the impedance of the mouthpiece and trumpet combined.

In order to compare TMM with FEM calculations, the transfer matrix elements are calculated as described in §2.B.i, and the full mouthpiece and trumpet impedance is then calculated as above.

3. IMPEDANCE COMPARISONS

A. PARAMETRIZATION OF MOUTHPIECE GEOMETRY

The parametrization of the mouthpiece shape used in this paper was introduced in Jackson and Scavone (2024). Figure 1 shows the parameters used. The throat is the narrowest part of the mouthpiece, and the l_{throat} parameter gives the length of the narrow part before the backbore expansion leading into the connection with the instrument. This parametrization matches well with the geometries of the manufactured mouthpieces for which the geometries have been measured, which are the Vincent Bach 5C and $1\frac{1}{2}$ C. We assume that other trumpet mouthpiece geometries can be adequately approximated with these parameters.

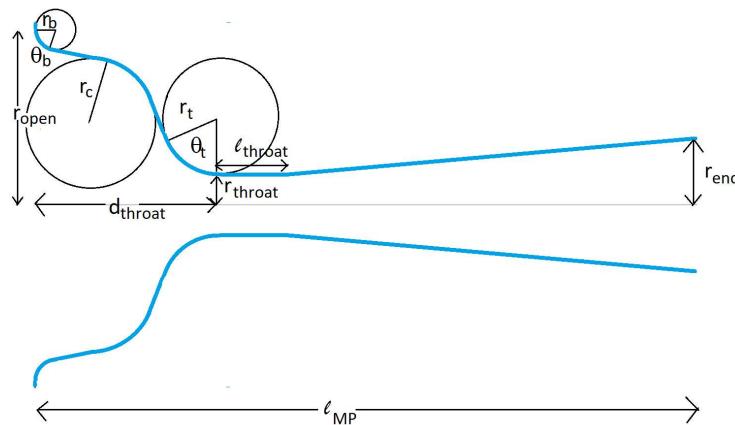


Figure 1: Parametrization of a trumpet mouthpiece.

It was shown in Jackson and Scavone (2024) that the calculated impedance of the parametrized geometry is very close to that of the measured geometry and to the measured impedance of the mouthpiece in question. We now compare, in further detail, impedances calculated using TMM and FEM to the measured impedances, including the locations of peak heights for both the impedance of the mouthpiece alone, and with the mouthpiece connected to a trumpet.

In addition, in this paper, we estimate the effect of modifying certain mouthpiece geometry parameters on the full instrument impedance. The results of this type of study could potentially be used by mouthpiece designers and manufacturers to determine which parameters should be modified in order to achieve a particular desired outcome in terms of playability.

B. FITTING MOUTHPIECE GEOMETRY TO MEASURED IMPEDANCE

Because the geometry of a mouthpiece or an instrument is not straightforward to accurately measure, it would be useful to be able to use an impedance measurement to estimate the geometry. In order to do this, we have fitted the mouthpiece geometry parameters to the impedance curve using TMM calculations. Because many calculations are required, the wide-pipe approximation is used. For this work, we have used a simple linear (the impedance magnitude is fit directly as a function of frequency) or logarithmic (the impedance is first expressed in dB and that is fit as a function of frequency) least-squares fit to the impedance curve, but future work will likely include an analysis of the properties of the impedance peaks.

The fit is performed with an objective of minimizing the least-squares difference between the calculated and measured impedance curves. The geometric parameters are varied in Matlab, and TMM is used to calculate the impedance curve corresponding to each resulting geometry, until a solution is found. For the fit, we use either the magnitude of the impedance as calculated or its logarithm. The latter would be expected to produce a result more relevant to the full impedance curve, rather than just to the first peak, which is by far the biggest in linear terms. The two fitting methods are compared.

C. COMPARISON OF THE MOUTHPIECE IMPEDANCE CALCULATION USING VARIOUS METHODS

The impedance of the 5C mouthpiece has been measured with a zProbe (Lefebvre and Scavone, 2011), a custom impedance probe, with a closed end. The impedance for this mouthpiece in the measurement configuration has been calculated with the following methods:

1. TMM calculation on the measured geometry with Bessel functions (see §2.A)
2. TMM calculation on the measured geometry with the “wide-pipe” approximation
3. TMM calculation based on the parametrized geometry (“wide-pipe”)
4. FEM calculation on the measured geometry using a velocity impulse at the inlet as input (see §2.B)
5. FEM calculation on the measured geometry using an acceleration at the inlet as input
6. FEM calculation on the measured geometry using a pressure impulse as input
7. TMM calculation based on the geometry fitted to the linear impedance (“wide-pipe”), for comparison with both the impedance calculated from the geometry and the measured impedance
8. TMM calculation based on the geometry fitted to the logarithmic impedance (“wide-pipe”), for a similar purpose as for 7. above

The impedance of the mouthpiece attached to a trumpet has also been measured. The impedance for this configuration is calculated by means of the method described in §2.C, and comparisons are made between the measurement and the calculation in each case.

D. VARIATION OF MOUTHPIECE GEOMETRY PARAMETERS

The parametrized geometry of the 5C mouthpiece has been altered by varying both the throat diameter and the cup depth separately, and leaving all the other parameters constant. The throat diameter of the original geometry ($2 \times r_{\text{throat}}$ in Fig. 1) is decreased by a total of 1.6 mm, in increments of 0.2 mm. It is likely that such a variation in throat diameter would have a significant effect on the playability aside from any consideration of impedance, just because of the change in the back pressure felt by the player, but it is a useful exercise for determining the acoustic effect.

Because cup depth is considered an important factor to trumpet players, the cup depth (d_{throat} in Fig. 1) of the 5C mouthpiece, which is considered to be moderate in comparison with other mouthpieces, is decreased by a total of 4 mm in increments of 0.5 mm. All other parameters remain the same as in the original 5C mouthpiece. This is done in order to determine whether the ease of playing high notes would increase. This would be apparent if the higher-frequency impedance peaks were to become more prominent, suggesting increased support from the air column.

4. RESULTS

A. ESTIMATION OF MOUTHPIECE GEOMETRY FROM IMPEDANCE

The results from the fits of the mouthpiece geometry from the impedance are shown in Figure 2. It is clear from the close-up on the right of Figure 2 that the linear fit does not yield an accurate geometry. However, the shape yielded by the logarithmic fit is qualitatively similar to the measured geometry, but there seems to be a discrepancy in the depth of the mouthpiece cup. This discrepancy is only about a millimetre and possibly due to the fact that the measurement plane of the probe is not completely defined, since the end of the probe protrudes slightly (a few tenths of a millimetre) into the object being measured. For a mouthpiece, which is measured to a fraction of a millimetre, the uncertainty in the location of the measurement plane is a serious issue that is planned to be resolved.

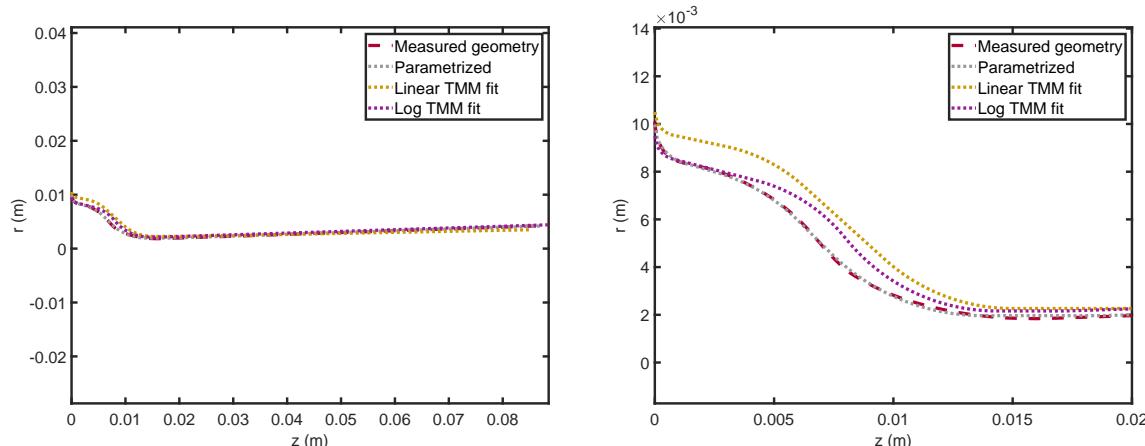


Figure 2: Fits of the geometry of the mouthpiece from an impedance measurement, as described in §3.B, as well as the measured and parametrized geometries. The graph on the right shows a close-up of the full mouthpiece graph on the left.

B. IMPEDANCE CALCULATIONS

The results of the TMM calculations for the geometries shown in Fig. 2 are shown in the left plot in Fig. 3. It is clear that the linear fit to the impedance is not sensitive to the regions not close to the first

impedance peak. The valley before the first peak is particularly poorly fit, because the fit is much more affected by the tip of the peak than by any other part of the curve.

The other impedance curves, including those calculated from FEM simulations, seem qualitatively similar. Therefore, a more detailed examination has been performed on the peak positions and heights. The results are shown in Table 1. From these, it is clear that there is no decisive winner among the various calculation methods. The calculation methods are further compared with regard to the properties of the full trumpet impedance in §4.C.

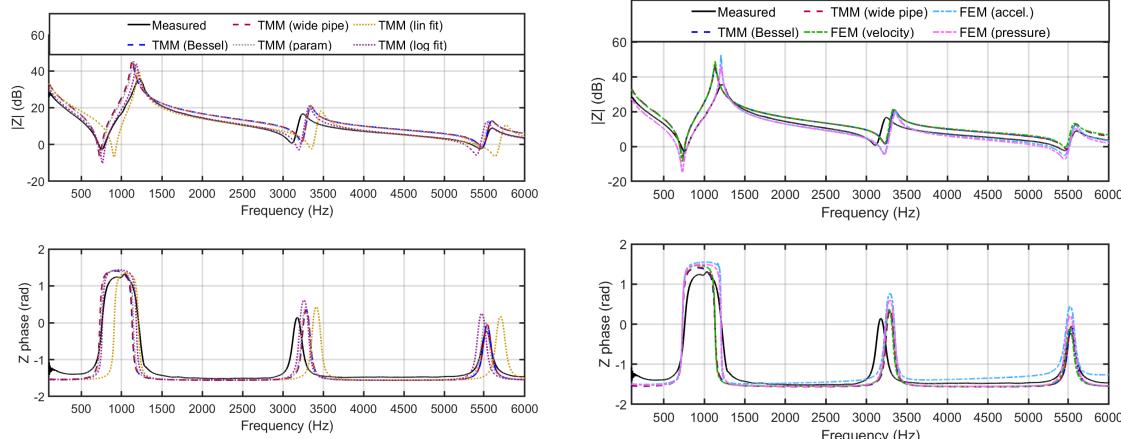


Figure 3: Impedance (upper panel) and phase (lower panel) of a measured impedance of a mouthpiece closed at the end, calculated impedances from TMM, and the result from a fit to the geometry of a mouthpiece (left), and the FEM fits with different inlet activations (right).

C. IMPLICATIONS FOR FULL INSTRUMENT IMPEDANCE AND PLAYABILITY

The same geometries and methods as in §4.B are used to calculate full trumpet impedance curves, for comparison with a measured impedance curve. This calculation is performed using the procedure described in §2.C. The results are shown in Fig. 4.

To apply these results to a factor of playability, the positions of the impedance peaks have been used to calculate inharmonicity curves. The goal is not to minimize the inharmonicity, but rather to best match the inharmonicity curve corresponding to the impedance measurement. The first peak has been ignored for this exercise, and the inharmonicity corresponding to each impedance curve is calculated by first calculating the mean ratio, $r_{\text{mean}} = \overline{f_n}/n$, for that impedance curve (where f_n is the frequency of the n th peak), then by calculating the deviation, in cents, of each peak frequency from the product $r_{\text{mean}}n$. The result of this calculation is shown in Fig. 5, and the RMS differences between the calculated and measured inharmonicities are listed in Table 2.

D. VARIATION OF MOUTHPIECE GEOMETRY PARAMETERS

The impedances of the mouthpiece and the full trumpet are calculated for different throat diameters and the result is shown in Fig. 6. The red curve corresponds to the original geometry of the 5C mouthpiece. Each curve represents a throat diameter 0.2 mm less than the last. It can be seen that the variation of the geometry affects the higher-frequency impedance peaks, whereas the lower-frequency ones remain largely unchanged. It is notable also that the relative peak heights remain largely unchanged with this variation in geometry.

Fig. 7 shows the impedances of the mouthpiece and the full trumpet for different cup depths. Again, the red curve corresponds to the original geometry of the 5C mouthpiece. Each curve represents a cup depth

Table 1: Mouthpiece impedance peak parameters. The best matches to the measured data (excluding the geometric fits) are indicated with boldface.

| | Peak 1 | | | | Peak 2 | | | | Peak 3 | | | |
|---------------------|-------------|--------------------|-------------|--------------------|-------------|--------------------|-------------|--------------------|-------------|--------------------|-------------|--------------------|
| | f (Hz) | Δf (Hz) | Z (dB) | ΔZ (dB) | f (Hz) | Δf (Hz) | Z (dB) | ΔZ (dB) | f (Hz) | Δf (Hz) | Z (dB) | ΔZ (dB) |
| Meas. | 1216.9 | — | 35.5 | — | 3426.5 | — | 16.7 | — | 5607.1 | — | 8.9 | — |
| TMM (Bessel) | 1132.3 | -124.7 | 45.0 | +9.5 | 3346.1 | +52.3 | 20.9 | +4.2 | 5597.9 | -2.8 | 12.8 | +3.9 |
| TMM (wide pipe) | 1133.4 | -123.1 | 45.8 | +10.3 | 3346.1 | +52.3 | 21.1 | +4.4 | 5597.9 | -2.8 | 12.8 | +3.9 |
| TMM (param) | 1148.1 | -100.8 | 45.6 | +10.1 | 3341.3 | +49.9 | 21.4 | +4.7 | 5577.4 | -9.2 | 13.2 | +4.3 |
| FEM (velocity) | 1133.4 | -123.1 | 48.9 | +13.4 | 3331.8 | +44.9 | 21.5 | +4.8 | 5574.1 | -10.2 | 13.4 | +4.5 |
| FEM (accel.) | 1205.9 | -15.7 | 52.6 | +17.1 | 3351.9 | +55.3 | 20.3 | +3.6 | 5587.3 | -6.1 | 12.3 | +3.4 |
| FEM (pressure) | 1204.8 | -17.3 | 46.5 | +11.0 | 3355.6 | +57.2 | 19.0 | +2.3 | 5590.9 | -5.0 | 10.3 | +1.4 |
| TMM (linear fit) | 1211.4 | -7.8 | 40.2 | +4.7 | 3375.0 | +117.8 | 18.3 | +1.6 | 5769.7 | +49.5 | 10.6 | +1.7 |
| TMM (log fit) | 1183.6 | -48.1 | 44.0 | +8.5 | 3346.1 | +41.5 | 20.9 | +4.2 | 5537.1 | -21.7 | 12.3 | +3.4 |

Table 2: RMS deviations of the inharmonicities shown in Fig. 5 from that corresponding to the measured trumpet impedance.

| Type | RMS error (cents) |
|----------------------------|-------------------|
| TMM (Bessel) | 6.7 |
| TMM (wide pipe) | 7.3 |
| TMM (param) | 6.5 |
| FEM (velocity input) | 14.5 |
| FEM (acceleration input) | 27.8 |
| FEM (pressure input) | 8.2 |
| TMM (linear geometric fit) | 22.2 |
| TMM (log geometric fit) | 3.1 |

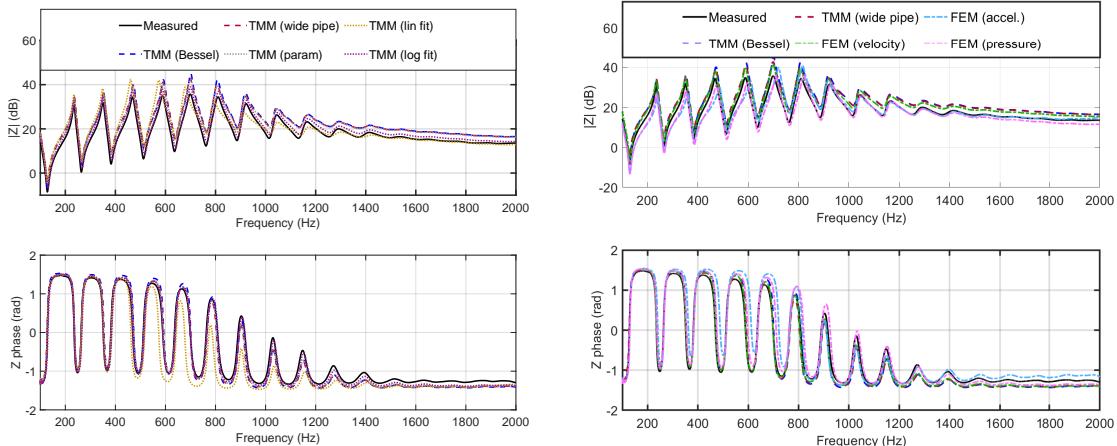


Figure 4: Impedance (upper panel) and phase (lower panel) from an impedance measurement of a full trumpet, calculated impedances from TMM and the result from a fit to the geometry of a mouthpiece (left), and the FEM fits with different inlet activations (right). The full trumpet impedances have been calculated using the method described in §2.C.

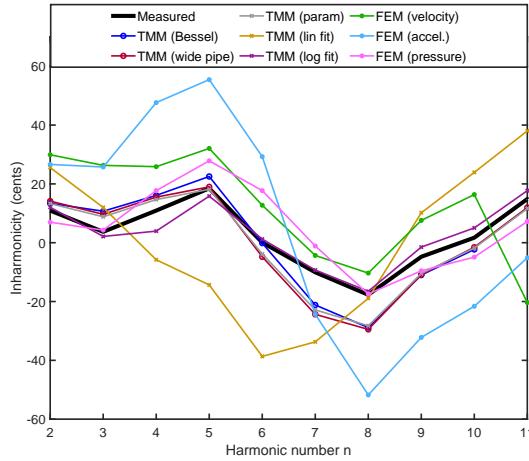


Figure 5: Inharmonicity curves corresponding to the impedances shown in Fig. 4.

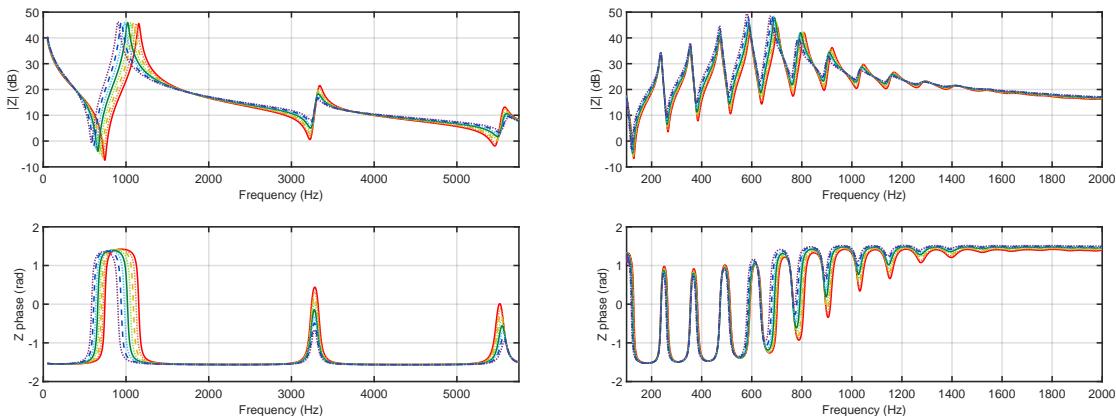


Figure 6: Impedance (upper panel) and phase (lower panel) as a result of incrementally varying the throat diameter by -0.2 mm. The mouthpiece impedances are given on the left, and the red curve represents the widest throat diameter. The full trumpet impedances (right) have been calculated using the method described in §2.C.

0.5 mm less than the last. As before, the lower-frequency peaks remain nearly unchanged, but there is a notable change in the relative peak heights of the higher-frequency peaks. They are much more pronounced

than for the original geometry, indicating that the higher notes corresponding to the peaks would likely be easier to play because the player would have more support from the air column. This correlates with the experience of trumpet players and instrument makers that a shallower mouthpiece is favoured by lead trumpet players who tend to play in the higher register, while a deeper mouthpiece is known to produce a darker tone with less contribution from the higher harmonics (Poirson et al., 2005).

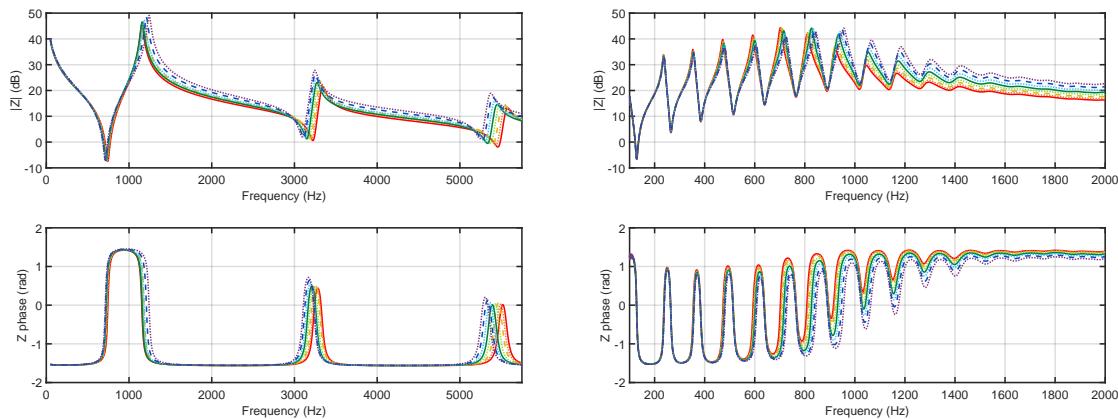


Figure 7: Impedance (upper panel) and phase (lower panel) as a result of incrementally varying the cup depth by -0.5 mm. The mouthpiece impedances are given on the left, and the red curve represents the deepest cup depth. The full trumpet impedances (right) have been calculated using the method described in §2.C.

5. CONCLUSION

In terms of extracting mouthpiece geometry parameters from an impedance measurement, as described in §3.B, from the results given in §4.A, it is clear that the logarithmic fit to the impedance has more potential to yield an accurate geometry than the linear fit. However, while the shape is roughly qualitatively reproduced, there is some deviation in the cup depth, which is possibly due to uncertainty in the location of the measurement plane. Given the small dimensions of the mouthpiece and the fact that the deviation is only a fraction of a millimetre, it will be necessary to ensure that the measurement plane is completely flat, without any protrusion into the object under measurement.

With regard to the list of calculation methods given in §3.C, it is not completely clear from the results given in §4.B which calculation method yields the best match to the measured impedance of a mouthpiece alone. However, the inharmonicity results for the full trumpet with mouthpiece given in §4.C indicate that a TMM calculation using Bessel functions may be the most accurate way to match the measured full trumpet impedance. However, the wide-pipe approximation is nearly as accurate, in this case. For this, we do not consider the geometric fits, because they do not represent calculations of impedance from geometry.

In the same section, of the calculations using FEM, it is clear that the one involving a pressure impulse at the inlet yields the most accurate match to the measured impedance, by far, compared with the other inlet activations. The likely reason for this is that a pressure impulse is most likely to give rise to a simple plane wave, whereas a velocity or acceleration impulse may give rise to higher order modes that are present in neither the measurement system nor the assumptions involved in the TMM calculation. Additionally, the accuracy of this method is comparable to that of the TMM calculation. This indicates that the pressure impulse is the best excitation method to use in FEM simulations when performing these types of impedance calculations.

The parametrization of the trumpet mouthpiece, as described in §3.D and the results of which are given in §4.D, allows for a determination of the effect on the playability on the variation of one or more parameters. For example, it is clear that a shallower mouthpiece would facilitate the production of notes in the high

register, which confirms the knowledge of instrument makers and trumpet players. To understand the more subtle effects on playability of varying the other mouthpiece geometry parameters, it will be necessary to perform playing simulations.

ACKNOWLEDGMENTS

This research was supported by an NSERC CGS funding award and by the Centre for Interdisciplinary Research in Music Media and Technology (CIRMMT). We are grateful to Cristian Bosc for useful mouthpiece data.

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