Modeling Wind Instrument Sound Radiation using Digital Waveguides

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Abstract

This paper develops the theory and digital waveguide implementation for modeling three-dimensional (3D) sound emission from wind instrument air columns. It is shown that the current acoustic theory regarding sound radiation from ducts and holes can be implemented in the digital waveguide context using properly designed digital filters. Each radiating sound source or hole requires a first- or second-order digital filter to account for angular- and frequency-dependent pressure distribution characteristics. Sound propagation delay from the source to the pickup is modeled with a delay line and possibly a fractional-delay interpolation filter. An additional digital filter to model attenuation in free space can also be used. The results of this model compare well with frequency-domain polar radiation calculations. A simplified system appropriate for real-time synthesis is developed that allows continuous pickup movement within an anechoic 3D space.

1 Motivation

Digital waveguide (DW) techniques have reached a high level of development with respect to wind instrument air columns, offering accurate and efficient means for implementing time-domain models which include toneholes and flared-bells. However, primary focus has previously been on modeling the internal acoustic behavior of the air column, and external direction-dependent sound radiation characteristics of the instrument have been largely neglected. That is, most current models implement either a virtual pickup somewhere inside the instrument air column or, for a slight improvement, apply a simple high-pass filter to the internal sound pressure to mimic transmission across the open-end junction. While this last technique represents a fairly accurate representation of the sound transmitted in a one-dimensional sense out of the bell, it does not offer any angular dependence with respect to arbitrary pickup points external to the instrument. This paper demonstrates the means for modeling such location-dependent sound radiation.

A further motivation for this work is the recent proliferation of three-dimensional virtual reality computer applications. The model presented here can be used within a 3D room simulation to provide a more realistic source signal.

This work also has developed as a natural extension to the efficient tonehole implementation previously reported (Scavone and Cook, 1998).

2 A Simple Wind Instrument Model

A simplified clarinet and its corresponding digital waveguide model are schematically represented in Fig. 1. The three functional components of the clarinet – its single-reed mouthpiece, uniform air column, and open end discontinuity – are modeled in the DW context by a non-linear excitation function, a bi-directional digital delay line, and a digital reflection filter, respectively. Sound pressure transmission across the open-pipe end discontinuity can be implemented without an additional filter as indicated, by assuming continuity of pressure. This is also equivalent to calculating the physical pressure at the open end of the pipe as the sum of the incoming and outgoing pressure traveling-wave components at the junction.

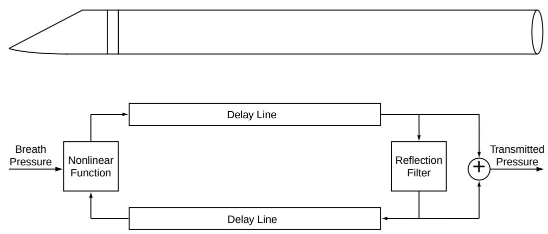

Figure 1: A simplified clarinet and its DW counterpart.

Levine and Schwinger (1948) developed the theory for pressure reflection at the end of an unflanged cylindrical pipe. Using their results, frequency-dependent sound pressure reflectance and transmittance characteristics for a pipe of 15 *mm* radius are shown in Fig. 2. Also shown in the figure are the magnitude responses of second-order digital filters designed to meet the Levine and Schwinger results. The filter order necessary for a given model is dependent on the pipe radius and the desired sampling rate. In general, reflection filters for open-pipe discontinuities display low-pass characteristics, while the complimentary transmission filters display high-pass characteristics.

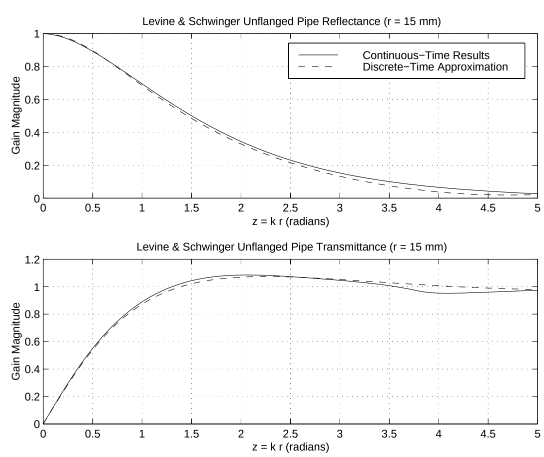

Figure 2: Sound reflection/transmission at the open end of a cylindrical pipe.

In this way, it is easy to determine the physical sound pressure at the open end of a cylindrical pipe. However, no information is provided to indicate how sound radiates into the three-dimensional space outside the instrument after it exits the pipe.

3 Sound Directivity for Open Pipes

Levine and Schwinger (1948) also provide results for the angular distribution of sound emission at the end of an open cylindrical pipe. Figure 3 displays these results as a function of frequency and angle relative to the main axis of the pipe. Because the directivity is symmetric about the pipe axis, only the range from $0° - 180°$ is shown. The Levine and Schwinger results are defined relative to an isotropically radiating point source and must be scaled appropriately. As should be expected, low-frequency sounds radiate from the open pipe in a nearly omni-directional pattern. High-frequency sounds are more direction-dependent, having greatest magnitude concentration along the axis and in front of the pipe opening.

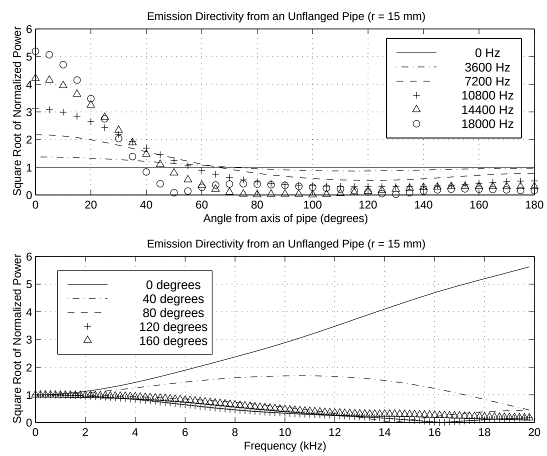

Figure 3: Sound emission directivity at the open end of a cylindrical pipe.

For implementation in the DW context, it is necessary to design low-order digital filters to match the desired directivity angles and frequency ranges. All calculations for this study were made using *MATLAB*® and the function *invfreqz*. In general, first- or second-order filters were sufficient for these purposes. When the pickup point is located directly in front of the pipe, the corresponding directivity filter applied to the physical pressure at the pipe opening has a high-pass characteristic. For pickup points to the side or behind the pipe opening, the directivity filter has a low-pass characteristic.

Use of the Levine and Schwinger results assumes an unflanged cylindrical pipe geometry. This assumption is appropriate for flutes and to lesser degree clarinets. However, such an assumption breaks down with respect to flared bells. Other studies have developed means for modeling the reflection characteristics of flared bells (Berners and Smith, 1994). Such responses typically require digital filters of higher order to accurately implement. As an approximation, a directivity filter can be designed based on the outer bell radius and applied to the bell output pressure determined using a "flared-bell" reflection filter.

4 Tonehole Radiation

DW implementations of toneholes have been previously discussed by Välimäki *et al.* (1993); Scavone and Smith (1997); Scavone and Cook (1998). The open-tonehole end can be reasonably approximated as an open-pipe discontinuity, with varying degrees of flanged to unflanged behavior. Thus, the models of sound transmission and radiation directivity previously discussed can be directly applied to the tonehole as well. It is only necessary to determine the physical pressure at the open-tonehole end and then filter that by an appropriately designed directivity filter.

5 Arbitrary External Pickup Points

The final stage in piecing together a radiation model is to choose an appropriate three dimensional coordinate system. An individual pickup point can then be implemented by summing together the radiated sound components from the various sound sources. This involves keeping track of the individual propagation delay and attenuation for each source, as diagrammed in Fig. 4. Lossless one-dimensional sound propagation in free space can easily be accounted for using digital delay lines. For the simplified instrument of Fig. 1 with only a single sound source, three dimensional radiation capabilities require the addition of a single second-order directivity filter, a delay line, and a fractional delay interpolation filter. Free space frequency-dependent attenuation can also be accurately accounted for using an additional digital filter designed according to the given propagation delay and the environmental variables.

The addition of multiple radiating sources with different angular orientations quickly complicates the implementation of 3D radiation. In theory, each radiating source requires an additional directivity filter, delay line, and delay interpolation filter.

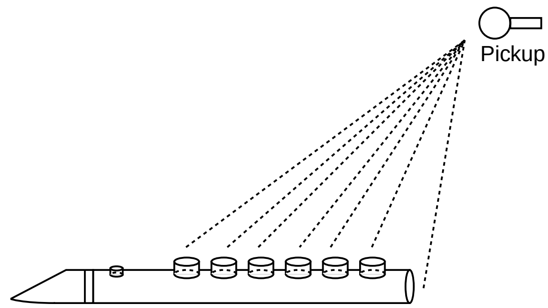

Figure 4: A clarinet model with a multiple radiating sound sources.

6 Results

A previous study of wind instrument directivity patterns was conducted by Rousseau (1996). Using frequency-domain techniques, together with far-field assumptions, Rousseau calculated directivity patterns for flute and oboe air columns. Rousseau then conducted experimental measurements in an anechoic chamber to verify the validity of the model and found good correspondence between the theoretical and measured data.

The results of the time-domain DW implementation can be compared to the frequency-domain calculations of Rousseau. Figure 5 shows such a comparison for an experimental cylindrical air column with six finger holes. Over varying fingering configurations and frequency ranges, the DW results match well those of the frequency-domain calculations. Discrepancies between the results can be attributed to different tonehole models, as well as fractional delay filtering effects and neglected internal air-column thermoviscous losses.

7 Realtime Efficiency Issues

Several approximations can be made to allow a more efficient implementation of a multiple tonehole instrument. Under a far-field assumption, all holes with similar orientation, as is the case for the toneholes in Fig. 4, can use the same directivity filter. That is, at a large distance from the toneholes, the angles from each source to the pickup are nearly equal. Under such a simplification, it is necessary to first use a "multi-input" tap delay line to sum together all the radiating pressure components with the correct delay offsets. The single directivity filter is then applied to the output of this delay line. Finally, fractional delay propagation distances can be neglected in the far-field with little loss of accuracy.

The model of Scavone and Cook (1998) allows dynamic tonehole opening and closing in a realtime implementation, through the use of a three-port tonehole junction and an allpass filter tonehole-end reflectance. While this open-pipe model is less accurate at high frequencies than the Levine and Schwinger model, directivity pattern calculations and comparisons show similar results.

Implementing a virtual pickup whose position can be smoothly modified in real time presents a number of digital signal processing complexities. In such a scenario, movement of the pickup implies constantly changing directivity filters, which necessitates a means for smoothly modifying filter coefficients. Similarly, real-time delay-line length changes need to be carefully handled to avoid "clicks". A detailed discussion of solutions to these problems is left to a future paper.

8 Acknowledgements

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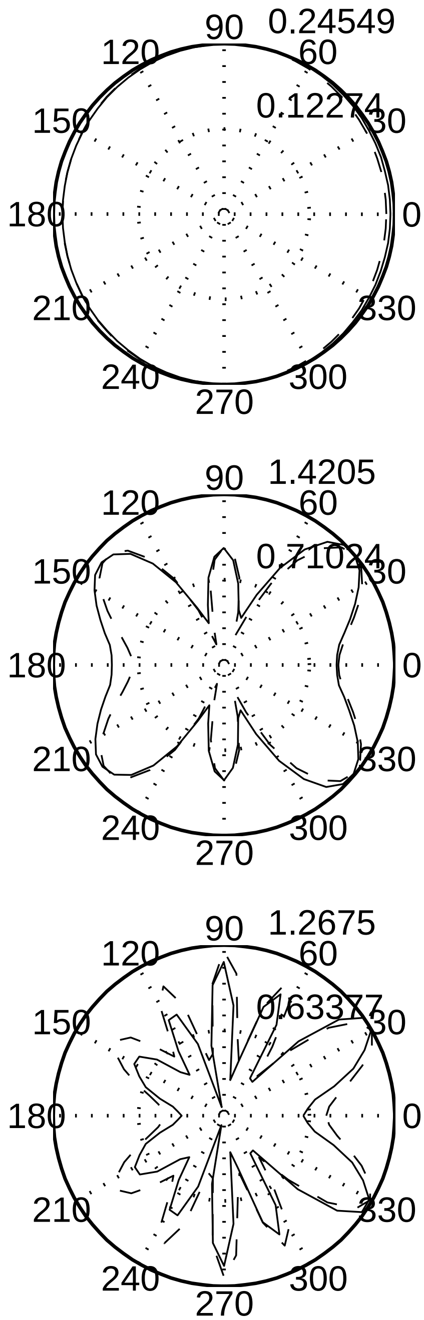

Figure 5: A comparison of the DW results (–) to the frequency-domain calculations (- -) of Rousseau (1996) for a six hole air column (three finger holes open, three finger holes closed). (top) 506 Hz; (middle) 2842 Hz; (bottom) 4845 Hz.