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3aMU3. Computational analysis of the dynamic flow in single-reed woodwind instruments

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The dynamics of the air flow within the mouthpiece of single-reed wind instruments plays an important contribution to the acoustic behavior of this type of systems, particularly during transient regimes. In this work, a two-dimensional numerical model of the mouthpiece-reed system is used to evaluate the behavior of the flow within the mouthpiece at two different frequencies within the playing range of the clarinet. The relationship between volume flow and blowing pressure, as well as the behavior of the vena contracta along one duty cycle are compared with the available analytical models and with recent experimental observations.

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INTRODUCTION

Studies on the acoustics of single-reed woodwind instrument have been conducted for more than a century [1] and the primary mechanisms of sound production in this family of instruments are well understood. However, many significant questions still remain to be answered, such as the importance of the geometrical aspects of the mouthpiece-reed system on the spectral content and dynamic behavior of the sound production, particularly during transients [2, 3].

Most theoretical models describe the relationship between the volume flow and the pressure difference across the mouthpiece channel by means of the Bernoulli obstruction theory for stationary flows [4, 5, 6, 7, 8]. Moreover, the flow behavior is assumed to be quasi-stationary, that is, the flow in a mouthpiece with an oscillating reed is assumed to be equal, at any instant, to the flow in a mouthpiece with a fixed reed having the same geometric configuration. Nevertheless, the Bernoulli obstruction theory for stationary flows holds only for cases where the influence of fluid viscosity is not significant, that is, when the Reynolds number $Re = U/h\nu$ is sufficiently high, where U is the volume flow and h and ν the height of the reed channel and the kinematic viscosity of the fluid, respectively. However, this is not the case when the reed is near to closure.

A more complex semi-empirical model considering viscous phenomena and assuming the quasi-stationary behavior has been proposed by Hirschberg et al. [9]. In their experimental investigations using a two-dimensional Borda tube, they noticed that two patterns of flow may occur simultaneously for Reynolds numbers $Re > 10$, depending on the ratio l/h , where l is the length of the channel. The flow remains fully detached for short channels ($l/h \leq 1$), whereas for long channels ($l/h \geq 3$) the flow reattaches at a roughly fixed point, l_r , measured from the channel's entrance. They also observed that, in the case of short channels, the vena contracta factor $\alpha = T_j/h$ was approximately constant with a value 0.6, where T_j is the thickness of the jet formed at the detached portion of the flow. van Zon et al. [10] found the same behavior using an idealized prototype of the mouthpiece with a static reed and assuming the flow to be two dimensional. They derived a more sophisticated flow model in which the transition between fully separated jet, described by the Bernoulli theory, to the attached jet, described by a Poiseuille flow, is represented by a boundary layer solution. Thus for short reed channels ($l/h \leq 1$) the flow is given by

$$U = \alpha h w \sqrt{\frac{2|\Delta p|}{\rho}} \operatorname{sgn}(\Delta p). \quad (1)$$

where w is the width of the reed channel, α is the vena contracta factor and Δp the pressure difference across the reed channel.

For long reed channels ($l \geq h3$), the flow is given by

$$U = \frac{12\nu w(l-l_r)(1-\delta^*)^2}{h(24c-1)} \left[1 - \sqrt{1 - \frac{h^4(24c-1)\Delta p}{72\rho\nu^2(l-l_r)^2(1-\delta^*)^2}} \right] \quad (2)$$

where ρ is the undisturbed density of the fluid, $\delta^* = 0.2688$ is a generalization of the boundary layer thickness for an arbitrary h , and $c = 0.0159$.

Other stationary measurements using realistic mouthpieces have found the same flow behavior, such as those conducted by Valkering [11] and by Dalmont et al. [12] in the case of the clarinet, and by Maurin [13] in the case of saxophones. However, previous attempts to investigate the behavior of the flow for a dynamical regime, that is, when the reed is free to oscillate, have provided results that do not correspond to those observed for a static reed [10, 14, 12], particularly for the behavior of the vena contracta factor. More recently, da Silva et al. [15] used the lattice Boltzmann method to investigate the dynamical behavior of the vena

contracta factor for mouthpiece reed systems of different geometries and found that, for long channels, the vena contracta remains constant for less than 30% of the duty cycle, increasing to about 40% of the duty cycle for short reed channels. This later result agrees with the experimental observations of Lorenzoni and Ragni [16] for a real saxophone mouthpiece in the dynamic regime using the particle image velocimetry technique. One important limitation in the investigations conducted by da Silva et al. [15] comes from the fact that the movement of the reed was induced by the flow instabilities, and therefore the oscillation frequency was much higher than that observed in real playing conditions.

The objective of this work is to investigate the dynamic behavior of the vena contracta factor at more realistic playing conditions. That includes different playing frequencies (200 and 1000 Hz), different dynamic regimes (loud and soft) and different reed channel lengths (short and long). The investigations are conducted by using the lattice Boltzmann method to represent a two-dimensional model. This paper is structured as follows: Section 2 describes the numerical method used in the investigations, the characteristics of the reed-mouthpiece model and the approach used to drive the reed oscillation at normal playing conditions. Section 3 presents the results for different playing conditions for short and long channels and discusses the limitations of the analysis. Finally, Section 4 provides a discussion of the results and suggestions for further improvement of the analytical model.

NUMERICAL PROCEDURE

The Lattice Boltzmann Method

The lattice Boltzmann method (LBM) differs considerably from the traditional computational fluid dynamic methods (CFD) based on the *continuum* theory, such as finite volumes, finite differences and so on. Instead of resolving the Navier-Stokes equations in order to determine macroscopic variables such as velocity and pressure, the LBM simulates the space-temporal evolution of a fluid-acoustic system based on a time-space discretization of the Boltzmann equation, known as the lattice Boltzmann equation (LBE). The Navier-Stokes and continuity equations can then be fully recovered from the LBE for low Mach numbers, namely ($Ma < 0.2$), by applying the Chapman-Enskog expansion, thus providing a physical validity for the method. Detailed descriptions of the LBM are provided by Succi [17] and Gladrow [18].

Further details of the LBM implementation used for this paper can be found in [15].

Mouthpiece-Reed Models

The models used in this work have already been discussed and thoroughly described [15]. Two different models of the mouthpiece-reed system based on a short and on a long reed channel are considered in this work, as depicted in Fig. 1. Both models are represented with a lattice grid containing 1002×252 cells, which corresponds to a lattice pitch $\Delta_x = 8 \times 10^{-5}$ m and a time step $\Delta_t = 1.3585 \times 10^{-7}$ s. The undisturbed fluid density was set equal to $\rho_0 = 1.0$ kg/m³ for convenience and the relaxation time was set to $\tau = 0.5495$, which implies a viscosity of $\nu' = 7.7657^{-4}$ m²/s, using $c_0 = 340$ m/s as the reference speed of sound.

The reed is represented by the model proposed by Avanzini and van Walstijn [19], which consists of a clamped-free bar of total length equal to 32.1×10^{-3} m and width $w = 10^{-2}$ m. However, instead of imposing a force component calculated from the pressure field around the reed, as described in Ref. [15], the reed movement is prescribed in terms of a harmonic reed displacement applied at its tip for two different frequencies, namely $f_0 = 200$ and 1000 Hz (the natural frequency of the reed is 1898 Hz). The interaction between the reed and the mouthpiece lay is assumed to be inelastic. This assumption is justified in Ref. [19]. The amplitude of the

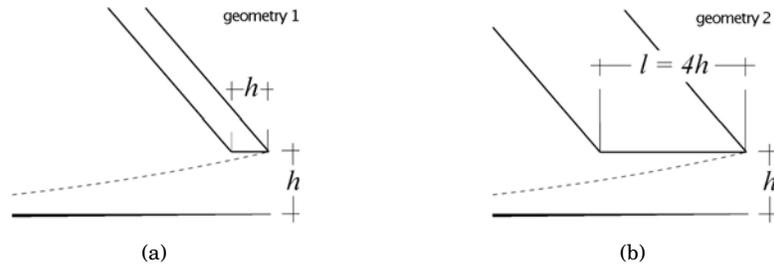


FIGURE 1: Channel profile geometries used in the simulation: (a) $L/h = 1$, and (b) $L/h = 4$.

reed displacement was specified as 1.44×10^{-3} and 0.6×10^{-3} m to simulate “soft” and “strong” (beating) dynamic range conditions, respectively (the equilibrium tip opening is 1.2×10^{-3} m and the prescribed displacement for the “strong” case clips at this distance). The simulation was run for a time duration necessary to allow the volume flow through the reed channel to reach a steady-state behaviour, which took less than two periods given the imposed reed displacement function. The results reported in subsequent sections were obtained from the third oscillatory period.

The algorithm assumes a no-slip condition of flow at the walls by implementing a bounce-back scheme, which inverts the direction of propagation of a distribution function f_i just before it reaches a solid boundary. This procedure creates a null fluid velocity at the walls and provides second-order accuracy to represent viscous boundary layer phenomena [20]. The moving boundary (the reed) within the lattice grid is implemented in terms of an interpolation scheme proposed by Lallemand and Luo [21]. This technique preserves second-order accuracy in representing the no-slip condition and the transfer of momentum from the boundary to the flow.

The mean flow is initiated by using a variation of the absorbing boundary conditions scheme proposed by Kam et al. [22], which is implemented by using a buffer between the fluid region and the open boundary to create an asymptotic transition toward a target flow. A desired feature of this technique is the anechoic characteristic that avoids any reflection or generation of spurious waves at the open boundaries.

The characteristics of the simulations conducted for the two geometries depicted in Fig. 1 are specified in Table 1.

TABLE 1: Specifications of the simulations cases investigated in the present work.

Geometry	Dynamic range	Frequency [Hz]
Short channel	soft playing	1000
		200
	strong playing	1000
		200
Long channel	soft playing	1000
		200
	strong playing	1000
		200

RESULTS

Dynamic Results for a Short Reed Channel

Comparisons between numerical and theoretical results for the volume flow through a short reed channel as a function of time for one duty cycle are depicted in Fig. 2. The theoretical values of flow are obtained from Eq. 1, using numerical values for Δp , h , and assuming α to be equal to 0.6, as observed in the experiments with a static reed. In general, the theoretical results are underestimated by approximately 17% when compared to numerical results. This is mainly due to the difference between experimental and numerical values of the vena contracta factor, as depicted in Fig. 3.

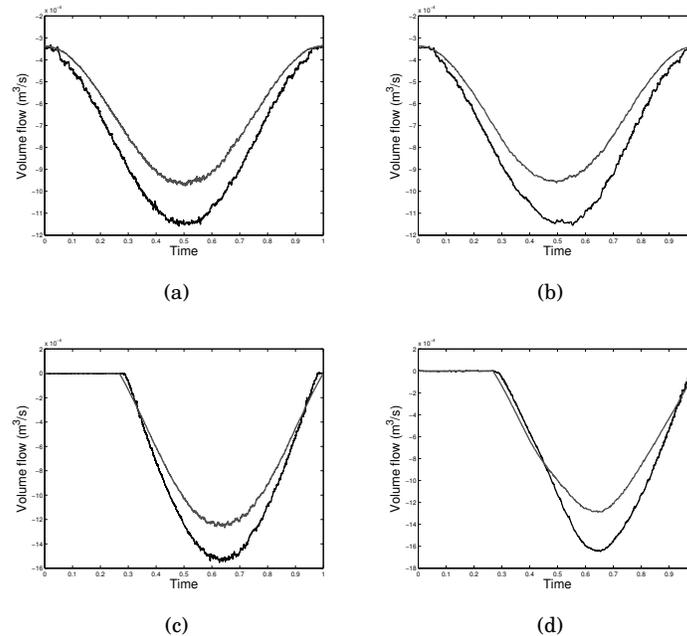


FIGURE 2: Comparison between numerical (black line) and analytical [10] (grey line) predictions for the volume flow for one duty cycle in a short reed channel ($L/h = 1$): (a) Soft playing, $f_0 = 200$ Hz, (b) soft playing, $f_0 = 1000$ Hz, (c) loud playing, $f_0 = 200$ Hz, and (d) loud playing, $f_0 = 1000$ Hz.

Interestingly, the vena contracta factor remains constant for most values of the modified Reynolds number $Re_1 = hRe/(L - h)$, as shown in Fig. 3. The average value of α seems to be proportional to the playing dynamic. For soft playing, the average α is around 0.7, whereas for loud playing the average value of α is about 0.78.

Dynamic Results for a Long Reed Channel

Comparisons between numerical and theoretical results for the volume flow through a long reed channel as a function of time for one duty cycle are depicted in Fig. 4. The theoretical values of flow are calculated with Eq. (2), assuming the reattachment point to be $l_r = 2h$, where h is the time-varying height of the channel aperture measured at the reed tip. In general, the agreement between analytical and numerical results are considerably better than that found in Ref. [15], although a maximum error of 22% can still be noticed for loud playing and low frequency conditions (Fig. 4(c)).

An analysis was conducted in order to investigate the instability of the reattachment point l_r . In order to do that, l_r was obtained from Eq. (2) using the numerical values of U , h and Δp .

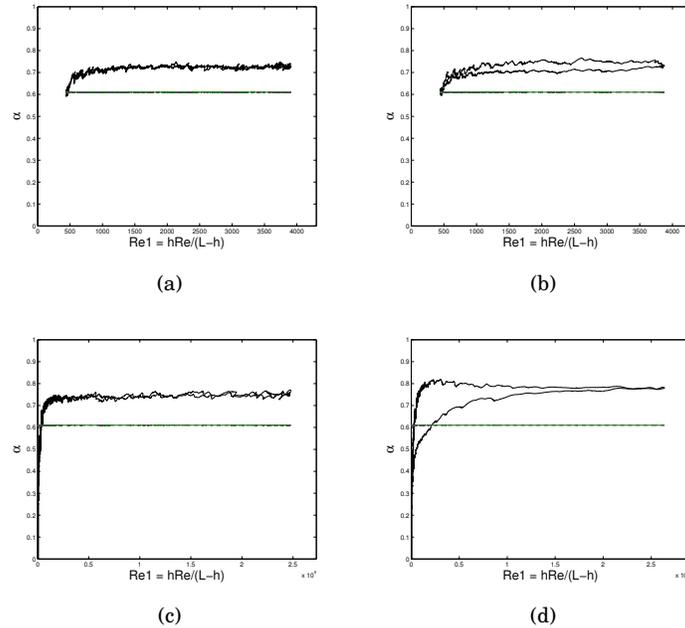


FIGURE 3: Comparison between numerical (black line) and analytical [10] (grey line) predictions for the vena contracta factor as a function of the Modified Reynolds number for one duty cycle and a short reed channel ($L/h = 1$): (a) Soft playing, $f_0 = 200$ Hz, (b) soft playing, $f_0 = 1000$ Hz, (c) loud playing, $f_0 = 200$ Hz, and (d) loud playing, $f_0 = 1000$ Hz.

Interestingly, the positions of the reattachment point were found at a distance much further from the mouthpiece entrance than predicted by the theory. In fact, for all cases the reattachment point took place at a distance greater than the reed channel length, which implies that the Poiseuille flow does not exist and the flow remains detached into the mouthpiece.

The graphics depicted in Fig. 5 indicate that the vena contracta factor remains nearly constant for most values of the modified Reynolds number $Re1 = hRe/(L - h)$, except for the limit of low Reynolds values, when the reed approaches complete closure and the viscous phenomena become significant. An exception to this behavior is observed in Fig. 5(d) for loud playing and 1000 Hz. The hysteretic behavior is explained by the fact that the reed opens due to a prescribed harmonic movement instead of being driven by the pressure difference Δp . This effect is particularly significant at high frequencies and constitutes a shortcoming of the present approach.

CONCLUSIONS

The behavior of the volume flow and vena contracta factor of a single-reed mouthpiece were investigated using the lattice Boltzmann method. The investigations were conducted for different playing conditions involving loud and soft playing, low and high notes, as well as short and long reed channel lengths.

For short reed channels, the numerical values of volume flow were found to be significantly higher than those found in experiments with a static reed [10, 9]. However, this result is in agreement with the experimental observations of Dalmont et al. [12] and Lorenzoni and Ragni [16].

For long reed channels, the numerical values of volume flow were approximately 22% lower than the values estimated by the theory [10]. An analysis conducted to evaluate the position of

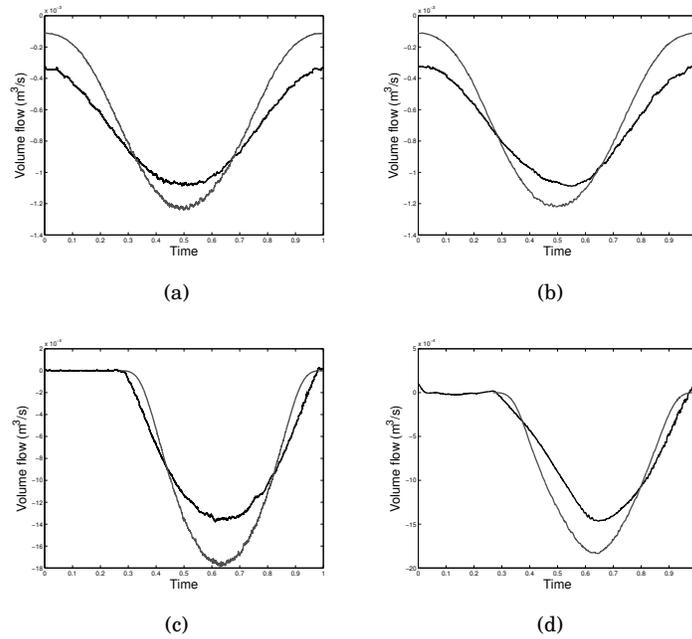


FIGURE 4: Comparison between numerical (black line) and analytical [10] (grey line) predictions for the volume flow for one duty cycle in a long reed channel ($L/h = 4$): (a) Soft playing, $f_0 = 200$ Hz, (b) soft playing, $f_0 = 1000$ Hz, (c) loud playing, $f_0 = 200$ Hz, and (d) loud playing, $f_0 = 1000$ Hz.

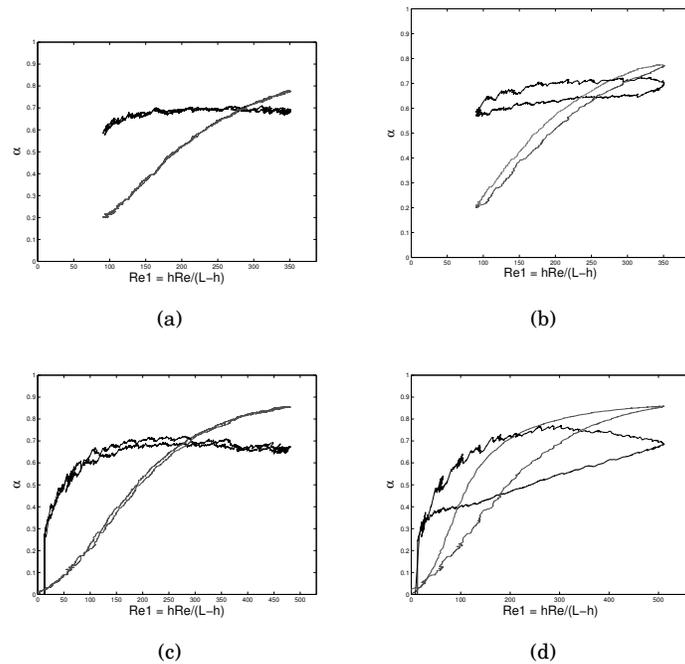


FIGURE 5: Comparison between numerical (black line) and analytical [10] (grey line) predictions for the vena contracta factor as a function of the Modified Reynolds number for one duty cycle and a long reed channel ($L/h = 4$): (a) Soft playing, $f_0 = 200$ Hz, (b) soft playing, $f_0 = 1000$ Hz, (c) loud playing, $f_0 = 200$ Hz, and (d) loud playing, $f_0 = 1000$ Hz.

the reattachment point found that, in all cases, $l_r > l$, which implies that the jet formed at the entrance does not reattach on the reed walls within the reed channel. Furthermore, the vena

contracta remains nearly constant, except for the limit of low Reynolds numbers when viscous phenomena begin to dominate the flow behavior.

The numerical technique presented in this work can be further explored to develop an improved analytical model for the flow into the mouthpiece as a function of the blowing pressure. However, the limitation of the method associated with the spurious hysteretic behavior caused by the prescribed movement of the reed must first be resolved.

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