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Power-balanced simulation of an ancient Paracas whistle

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Abstract:	In collaboration with the Museum of Fine Arts, Boston, X-ray computed tomography (CT) scans were made of a ceramic whistle from the Paracas culture (600–175 BC). A repaired crack in the whistle, along with the ceramic's sensitivity to moisture, prevents the instrument from being played. By scanning the whistle, it was possible to produce 3D-printed replicas that can be handled and played. These replicas can also be measured using an impedance head to quantify the internal acoustic admittance and mouth radiation impedance of the whistle. From these measurements, a lumped power-balanced model was derived using the port-Hamiltonian system framework and numerically simulated using finite-difference time-domain (FDTD) methods. These simulations were used to produce a real-time physics-based simulation capable of reproducing the sound of the whistle.
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1. INTRODUCTION

X-ray computed tomography (CT) scanning is a powerful tool for studying and preserving historical musical instruments. In this paper, we utilize the three-dimensional geometry collected from a CT scan to inform a physical modeling synthesis algorithm enabling the recreation of an instrument's sound. We examine and synthesize the sounds of a whistle from the Paracas culture (600-175 BC) that was located in what is now modern-day Peru.

The whistle under study is a jet-driven stopped pipe. A simplified model of the jet-drive excitation mechanism in flutes and recorder-like instruments was proposed by Coltman¹ and various refinements have been proposed since.²⁻⁵ The model presented in this paper is largely based on the semi-empirical model proposed by de la Cuadra⁵ with aspects inspired by findings on turbulent jets in stopped pipes described by Auvray et al.⁶ The excitation mechanism is connected to two passive systems, the whistle's body acoustic admittance and its mouth radiation impedance, using the port-Hamiltonian (pH) system framework.⁷ This enables a passive description of the entire system.

To characterize the body admittance and mouth radiation impedance, the CT scan data was used to produce two different 3D-printed replicas which isolate the whistle's internal and external geometry. Measurements of the 3D replicas were made using a custom impedance head.⁸ For validation, the measurement results were compared to 3D FEM simulations.

Section 2 describes a simplified physical model of the whistle including the jet-drive mechanism, the two passive pH system models, and how all sub-models are interconnected. Section 3 describes a discrete scheme for simulating the whistle's physical model. Section 4 describes the measurements used to characterize the whistle's body admittance and mouth radiation impedance. Section 5 discusses the results of a numerical simulation and Section 6 concludes the paper.

2. PHYSICAL MODELING OF THE FLUTE: A PORT-HAMILTONIAN APPROACH

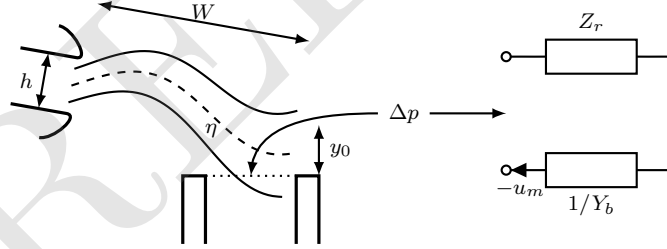


Figure 1: Left: Representation of the jet excitation generating the input pressure Δp . Right: Equivalent circuit with bore admittance Y_b and radiation impedance Z_r .

A simplified physical model of the flute can be broken down into three components: the nonlinear jet-drive mechanism, the linear body admittance, and the linear radiation impedance.

A. JET DRIVE EXCITATION MECHANISM

A diagram of the jet drive excitation is shown on the left of Figure 1. The general principle of the jet drive mechanism is that a transverse jet perturbation is formed at the exit of a flue or the player's mouth. The jet is convected towards a labium or edge while growing in size due to the instability of the jet. The interaction between the jet and labium is interpreted as a dipole source that produces an aeroacoustic force which excites the instrument. The interaction depends on the position of the centerline of the jet, $\eta(x, t)$. The jet centerline is affected by the volume flow at the instrument interface, resulting in a feedback mechanism. In stopped

pipe instruments, the jet is in crossflow as the instrument's flow recirculates out of the instrument.⁶ The jet displacement model presented here follows the semi-empirical model proposed by de la Cuadra,⁵ in which the transverse jet-displacement is described by

$$\eta(x, t) = \frac{he^{\alpha_i x}}{v_j(t)} \frac{v_m(t - x/c_p)}{S_m} + \langle \eta \rangle. \quad (1)$$

h is the channel height in meters, v_j is the jet velocity in m/s, S_m is the input area to the mouth of the instrument in m^2 , $v_m = u_m/S_m$ is the velocity of the flow through the entrance to the bore in m/s with u_m the volume velocity. $\alpha_i = \beta/h$ is the growth rate of the jet and $c_p = \gamma v_j$ is the phase velocity of the jet. β and γ are non-dimensional coefficients with no physical meaning and have been experimentally found to be $\beta \approx 0.3$ and $\gamma \approx 0.4$.^{5,6} At the labium, $x = W$, the jet-displacement is proportional to the mouth volume velocity delayed by $\tau = W/c_p$. $\langle \eta \rangle$ is the jet deflection due to the crossflow equal to,

$$\langle \eta \rangle = a_\delta \frac{x^2}{rh}, \quad r = \frac{v_j}{\langle v_m \rangle}. \quad (2)$$

a_δ is a tunable parameter and r is the ratio of the jet velocity to the mean crossflow velocity, $\langle v_m \rangle$.

The aeroacoustic force at the labium acts as a pressure step

$$\Delta p_j = -\frac{\rho \delta_d}{S_m} d_t Q = -\frac{\rho b \delta_d}{W} d_t \left[v_j(t) \left(1 + \tanh \left(\frac{y_0 - \eta(t)}{b} \right) \right) \right], \quad (3)$$

where ρ is the density of air in kg/m^3 and $\delta_d = 4\sqrt{2hW}/\pi$ is the effective acoustic distance between the dipole sources. Q is the portion of the jet entering the pipe,

$$Q = h \int_{-\infty}^{y_0 - \eta(t)} v(y) dy, \quad v(y) = v_j(t) \text{sech}^2 \left(\frac{y}{b} \right), \quad (4)$$

where y_0 is the offset of the centerline from the labium and $v(y)$ is the assumed Bickley jet velocity profile with $b = 2h/5$, in m, the half width as suggested by Nolle.⁹ $v_j(t)$ is not assumed to be slowly varying. An important contribution to the excitation mechanism model is modeling the nonlinear loss due to vortex shedding. Vortex shedding occurs due to flow separation at the labium which results in the formation of vortices.^{1,3,4} The kinetic energy dissipated in the formation of said vortices is modeled as a lumped nonlinear loss

$$\Delta p_l = \text{sign}(u_m) \frac{\rho}{2(\alpha_m S_m)^2} u_m^2, \quad (5)$$

where α_m represents a vena contracta factor. The last component of the input force is a turbulent noise source with amplitude³

$$\Delta p_t \propto -\frac{\rho}{2} v_j^2(t). \quad (6)$$

For the turbulent jet with crossflow considered here, this noise component is modified to have an amplitude

$$\Delta p_t \propto -\left(\frac{\rho}{2} \langle v_m \rangle^2 + a_t \right), \quad (7)$$

where a_t represents a constant noise source due to the turbulent nature of the jet, and a term in $\langle v_m \rangle^2$ is due to the crossflow. The total excitation pressure is

$$\Delta p = \Delta p_j + \Delta p_l + \Delta p_t. \quad (8)$$

B. PORT-HAMILTONIAN SYSTEMS

The pH system framework is a useful formalism for designing and analyzing passive systems.⁷ The formalism is based on the concept of energy flow, where the product of effort, $\mathbf{e}$, and flow, $\mathbf{f}$, variables is equal to power. In the acoustic domain, the flow and effort variables correspond to pressure, p , and volume velocity, u . Energy is stored in state variables that are the time integrals of the flow and effort variables. The state variable associated with pressure is ψ , the acoustic momenta per unit area (in $\text{kg}\cdot\text{m}^3/\text{s}$) and the state variable associated with volume velocity is v , the acoustic volume displacement (in m^3). For the purposes of this paper, the system is considered a pH system if it can be written as

$$\underbrace{\begin{bmatrix} d_t \mathbf{x} \\ \mathbf{y} \end{bmatrix}}_{\mathbf{f}} = \left(\underbrace{\begin{bmatrix} \mathbf{J} & \mathbf{G} \\ -\mathbf{G}^T & -\mathbf{M} \end{bmatrix}}_{\mathcal{J}} - \underbrace{\begin{bmatrix} \mathbf{R} & \mathbf{P} \\ \mathbf{P} & \mathbf{S} \end{bmatrix}}_{\mathcal{R}} \right) \underbrace{\begin{bmatrix} \nabla H(\mathbf{x}) \\ \mathbf{u} \end{bmatrix}}_{\mathbf{e}}. \quad (9)$$

$\mathcal{J} = -\mathcal{J}^T$ is a block skew-symmetric matrix and $\mathcal{R} = \mathcal{R}^T$ is a block symmetric matrix. This form is referred to as the input-state-output port-Hamiltonian system with feedthrough term [7, §4.1]. This form satisfies the following power balance,

$$d_t H(\mathbf{x}) = \mathbf{u}^T \mathbf{y} - \underbrace{\begin{bmatrix} \nabla H(\mathbf{x}) \\ \mathbf{u} \end{bmatrix}^T \mathcal{R} \begin{bmatrix} \nabla H(\mathbf{x}) \\ \mathbf{u} \end{bmatrix}}_Q, \quad (10)$$

and is passive under the condition that $\mathcal{R}$ is positive semi-definite and $H(\mathbf{x})$ is non-negative. pH systems can be interconnected without affecting the stability of the entire system.

C. MODAL BORE MODEL

The response of the bore is modeled using a modal representation of the input admittance with N modes, described by the pH system

$$d_t \begin{bmatrix} \psi \\ \mathbf{v} \end{bmatrix} = \left(\underbrace{\begin{bmatrix} 0 & -\mathbf{I} \\ \mathbf{I} & 0 \end{bmatrix}}_{\mathbf{J}_b} - \underbrace{\begin{bmatrix} 2\mathbf{R} & 0 \\ 0 & 0 \end{bmatrix}}_{\mathbf{R}_b} \right) \underbrace{\begin{bmatrix} \mathbf{M}^{-1}\psi \\ \mathbf{K}\mathbf{v} \end{bmatrix}}_{\mathbf{e}_b} + \underbrace{\begin{bmatrix} \mathbf{1} \\ 0 \end{bmatrix}}_{\mathbf{G}_b} p_b, \quad (11a)$$

$$u_b = \mathbf{G}_b^T \mathbf{e}_b. \quad (11b)$$

p_b is the pressure at the input to the bore and u_b is the corresponding volume velocity. ψ and $\mathbf{v}$ are both $\in \mathbb{R}^{N \times 1}$. $\mathbf{1} \in \mathbb{R}^{N \times 1}$ is a column vector of all ones and $\mathbf{I} \in \mathbb{R}^{N \times N}$ is the identity matrix. $\mathbf{M}$, $\mathbf{K}$, and $\mathbf{R}$, all $\in \mathbb{R}^{N \times N}$, are the diagonal modal mass, stiffness, and damping matrices in kg, N/m, and kg/s, respectively. The diagonal elements of each matrix are defined by

$$M_{m,m} = 1/g_m, \quad R_{m,m} = \zeta_m \omega_m, \quad K_{m,m} = \omega_m^2/g_m, \quad (12)$$

with ω_m the mode radian frequency in rads/s, g_m is the gain of each mode in kg^{-1} , and ζ_m is the mode damping ratio.

D. RADIATION MODEL

Rational and causal approximations of the radiation impedance for flanged and unflanged pipes were proposed by Silva et al.¹⁰ This form can be realized as a lumped circuit¹¹ and pH system¹²

$$d_t \begin{bmatrix} \psi_r \\ v_r \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{\mathbf{J}_r} - \underbrace{\begin{bmatrix} R & 0 \\ 0 & G \end{bmatrix}}_{\mathbf{R}_r} \underbrace{\begin{bmatrix} \psi_r/L \\ v_r/C \end{bmatrix}}_{\mathbf{e}_r} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{\mathbf{G}_r} - \underbrace{\begin{bmatrix} R \\ 0 \end{bmatrix}}_{\mathbf{P}_r} u_r, \quad (13a)$$

$$p_r = (\mathbf{G}_r + \mathbf{P}_r)^T \mathbf{e}_r + R p_r. \quad (13b)$$

The radiation circuit parameters are based on a general radiation impedance representation¹³ with the transfer function

$$\frac{Z_r}{Z_c} = \frac{b_2 (jka)^2 + b_1 (jka)}{a_2 (jka)^2 + a_1 (jka) + 1}. \quad (14)$$

b_1 , b_2 , a_1 , and a_2 are filter coefficients that can be determined from an optimization procedure. $k = \omega/c$ is the wavenumber with c the speed of sound in air. a is the radius associated with the radiation surface. If $a_2 = b_2$, in the high-frequency limit Z_r approaches Z_c . The radiation circuit parameters are equal to

$$R = Z_c \frac{a_2 b_1^2 + b_2^2 - a_1 b_1 b_2}{a_2 (a_1 b_1 - b_2)}, \quad G = \frac{a_2}{Z_c b_2}, \quad L = \frac{Z_c a}{c} b_1, \quad C = \frac{a}{Z_c c} \frac{a_2^2 b_1}{a_2 b_1^2 + b_2^2 - a_1 b_1 b_2}, \quad (15)$$

which have units $\text{kg}/(\text{m}^4 \cdot \text{s})$, $(\text{m}^4 \cdot \text{s})/\text{kg}$, $\text{kg}/(\text{m}^3)$, and $\text{s}/(\text{kg} \cdot \text{m})$, respectively, and are subject to the realizability conditions

$$a_2 b_1^2 + b_2^2 > a_1 b_1 b_2, \quad a_1 b_1 > b_2. \quad (16)$$

E. INTERCONNECTION AND ENERGY ANALYSIS

The three subsystems are related by the following interconnection matrix

$$\begin{bmatrix} u_m \\ p_b \\ u_r \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \Delta p \\ u_b \\ p_r \end{bmatrix}. \quad (17)$$

An energy analysis of the linear pH system components follows from an inner product of the inputs and outputs in Eq. (17) and the inner product of efforts and flows in Eqs. (11)–(13). The resulting power balance is

$$d_t (H_b + H_r) = P_{\text{jet}} + P_{\text{noise}} - (Q_l + Q_b + Q_r), \quad (18)$$

where H_b and H_r are the energy stored in the bore and radiation subsystems, respectively, and Q_b and Q_r are corresponding power dissipation terms.

$$H_b = \frac{\psi^T \mathbf{M}^{-1} \psi}{2} + \frac{\mathbf{v}^T \mathbf{K} \mathbf{v}}{2}, \quad Q_b = 2 (\mathbf{M}^{-1} \psi)^T \mathbf{R} \mathbf{M}^{-1} \psi \quad (19a)$$

$$H_r = \frac{\psi_r^2}{2L} + \frac{v_r^2}{2C}, \quad Q_r = \begin{bmatrix} \mathbf{e}_r \\ p_r \end{bmatrix}^T \begin{bmatrix} \mathbf{R}_r & \mathbf{P}_r \\ \mathbf{P}_r^T & R \end{bmatrix} \begin{bmatrix} \mathbf{e}_r \\ p_r \end{bmatrix}. \quad (19b)$$

The terms H_b , H_r , Q_l , Q_b , and Q_r are all non-negative if the associated parameter values are non-negative. P_{jet} and P_{noise} are the power supplied by the jet and turbulent noise, respectively. Q_l is the power dissipated by vortex shedding:

$$P_{\text{jet}} = u_m \Delta p_j, \quad P_{\text{noise}} = u_m \Delta p_t, \quad Q_l = \frac{\rho}{2\alpha_m^2 S_m^2} |u_m|^3. \quad (20)$$

The total system is passive under the assumption that P_{jet} is interpreted solely as a power source and the energy stored in the jet delay is ignored.

3. FINITE DIFFERENCE SIMULATION

The system of equations presented in the prior section is simulated using finite-difference time-domain (FDTD) methods. We introduce the half shift operators, $e_{\pm}$, and the interleaved difference and average operators

$$e_{\pm}x(t) = x(t \pm \frac{1}{2}\Delta_t), \quad \delta_t = \frac{e_+ - e_-}{\Delta_t}, \quad \mu_t = \frac{e_+ + e_-}{2}, \quad (21)$$

where Δ_t is the sampling period in s. The following notation $x^n = x(n\Delta_t)$ is used to represent a discrete time series. These operators are second-order accurate at time n . Important properties of these operators are

$$\delta_t x^n (\mu_t x^n) = \delta_t \left(\frac{(x^n)^2}{2} \right), \quad \delta_t = \frac{2}{\Delta_t} (\mu_t - e_-), \quad \mu_t = \frac{\Delta_t}{2} \delta_t + e_-. \quad (22)$$

The first property helps form the discrete gradient method allowing one to realize the evolution of numerical energy exactly.

A. DISCRETIZATION

The jet-drive and vortex shedding equations are discretized as

$$\Delta p_j^n = \frac{\rho \delta_d h b}{S_m} \delta_t \left[v_j^n \tanh \left(\frac{y_0 - \eta^n}{b} \right) \right], \quad (23)$$

$$\Delta p_l^n = \text{sign}(u_m^n) \frac{\rho}{2\alpha S_m} (u_m^n)^2, \quad (24)$$

and η^n is discretized as

$$\eta^n = \frac{h e^{\alpha_i x} u_m^{n-\tau/\Delta_t}}{v_j^n + \varepsilon} \frac{1}{S_m} + \langle \eta \rangle^n, \quad (25)$$

where ε is machine epsilon used to prevent divide-by-zero errors. The delayed values of u_m^n/S_m are modeled using a time-varying delay line based on Lagrange interpolation¹⁴ with two read pointers for $n - \tau/\Delta_t + \frac{1}{2}$ and $n - \tau/\Delta_t - \frac{1}{2}$. To prevent a delay-free loop, the lowest realizable delay is $\tau = \Delta_t/2$. To compute the centerline deflection $\langle \eta \rangle^n$, the mean value of $\langle v_m^n \rangle = \langle u_m^n/S_m \rangle$ is computed using a root-mean-square level detector based on a leaky integrator.¹⁵ The modal bore is discretized as

$$\delta_t \psi^n = -\mathbf{K} \mu_t \mathbf{v}^n - 2\mathbf{R} \mathbf{M}^{-1} \mu_t \psi^n + \mathbf{1} p_b^n, \quad \delta_t \mathbf{v}^n = \mathbf{M}^{-1} \mu_t \psi^n, \quad (26a)$$

$$u_b^n = \mathbf{1}^T \mathbf{M}^{-1} \mu_t \psi^n. \quad (26b)$$

The modal damping and resonant frequency can be re-parameterized to account for frequency warping due to the discretization above as

$$\overline{\zeta_m \omega_m} = \frac{2}{\Delta_t} \frac{1 - r_m^2}{r_m^2 + 2w_m}, \quad \overline{\omega_m} = \frac{2}{\Delta_t} \sqrt{\frac{r_m^2 - 2w_m r_m + 1}{r_m^2 + 2w_m r_m + 1}}, \quad (27)$$

with $r_m = \exp(-\zeta_m \omega_m \Delta_t)$, and $w_m = \cos(\omega_m \Delta_t)$. The values for $\overline{\zeta_m \omega_m}$ and $\overline{\omega_m}$ can be substituted into Eq. (12) to derive the re-parameterized modal stiffness and damping. The radiation impedance is discretized

as

$$\delta_t \psi_r^n = -\frac{1}{C} \mu_t v_r^n - \frac{R}{L} \mu_t \psi_r^n - R u_r^n, \quad \delta_t v_r^n = \frac{1}{L} \mu_t \psi_r^n - \frac{G}{C} \mu_t v_r^n + u_r^n, \quad (28a)$$

$$p_r^n = \frac{R}{L} \mu_t \psi_r^n + \frac{1}{C} \mu_t v_r^n + R u_r^n. \quad (28b)$$

B. SCHEME UPDATE

The discretization of the bore, Eq. (26), and radiation, Eq. (28), results in a semi-explicit scheme. The state update and output equations are found by solving the linear system. For the bore, the resulting update is:

$$\psi^{n+\frac{1}{2}} = \mathbf{A}_{\psi\psi} \psi^{n-\frac{1}{2}} + \mathbf{A}_{\psi v} \mathbf{v}^{n-\frac{1}{2}} + \mathbf{B}_{\psi} p_b, \quad \mathbf{v}^{n+\frac{1}{2}} = \mathbf{A}_{v\psi} \psi^{n-\frac{1}{2}} + \mathbf{A}_{vv} \mathbf{v}^{n-\frac{1}{2}} + \mathbf{B}_v p_b, \quad (29a)$$

$$u_b^n = \mathbf{C}_{\psi} \psi^{n-\frac{1}{2}} + \mathbf{C}_v \mathbf{v}^{n-\frac{1}{2}} + d_b p_b^n. \quad (29b)$$

The update for the radiation scheme is

$$\psi_r^{n+\frac{1}{2}} = a_{\psi\psi} \psi_r^{n-\frac{1}{2}} + a_{\psi v} v_r^{n-\frac{1}{2}} + b_{\psi} u_r, \quad v_r^{n+\frac{1}{2}} = a_{v\psi} \psi_r^{n-\frac{1}{2}} + a_{vv} v_r^{n-\frac{1}{2}} + b_v u_r, \quad (30a)$$

$$p_r^n = c_{\psi} \psi_r^{n-\frac{1}{2}} + c_v v_r^{n-\frac{1}{2}} + d_r u_r^n. \quad (30b)$$

All the coefficients in Eqs. (29) and (30) can be computed prior to any simulations.

To resolve the nonlinearity associated with vortex shedding, the quantity $p_b^n = p_r^n + \Delta p^n$ from Eq. (17) is solved by substituting in p_r^n from Eq. (30b) and using the definition of Δp^n

$$p_b^n = c_{\psi} \psi_r^{n-\frac{1}{2}} + c_v v_r^{n-\frac{1}{2}} + d_r u_r^n + \Delta p_j^n + \Delta p_t^n + \text{sign}(u_m^n) \frac{\rho}{2\alpha^2 S_m^2} (u_m^n)^2. \quad (31)$$

The above expression is substituted into Eq. (29b) and, by equating $u_b^n = -u_r^n = -u_m^n$, the resulting quadratic in $|u_b^n|$ is found:

$$-c_2 \text{sign}(u_m^n) |u_m^n|^2 - c_1 \text{sign}(u_m^n) |u_m^n| + c_0^n = 0, \quad (32)$$

with

$$c_2 = d_b \frac{\rho}{2\alpha^2 S_m^2}, \quad c_1 = 1 + d_b d_r, \quad (33a)$$

$$c_0^n = \mathbf{C}_{\psi} \psi^{n-\frac{1}{2}} + \mathbf{C}_v \mathbf{v}^{n-\frac{1}{2}} + d_b \left(c_{\psi} \psi_r^{n-\frac{1}{2}} + c_v v_r^{n-\frac{1}{2}} - (\Delta p_j^n + \Delta p_t^n) \right). \quad (33b)$$

The coefficients $c_2, c_1 > 0$, and the positive solution for $|u_b^n|$ is found from

$$|u_b^n| = \frac{-c_1 + \sqrt{c_1^2 + 4c_2 |c_0^n|}}{2c_2}, \quad \text{sign}(u_b^n) = \text{sign}(c_0^n). \quad (34)$$

Now that u_b^n has been determined, the complete expression for Δp^n is found by computing Eq. (24). It is then possible to update the radiation system states and solve for p_r^n using Eqs. (30). p_b^n results from $\Delta p^n + p_r^n$ which is finally used to update the modal states of the bore. The resulting simulation is lightweight and is capable of real-time computations when implemented using the C++ language.

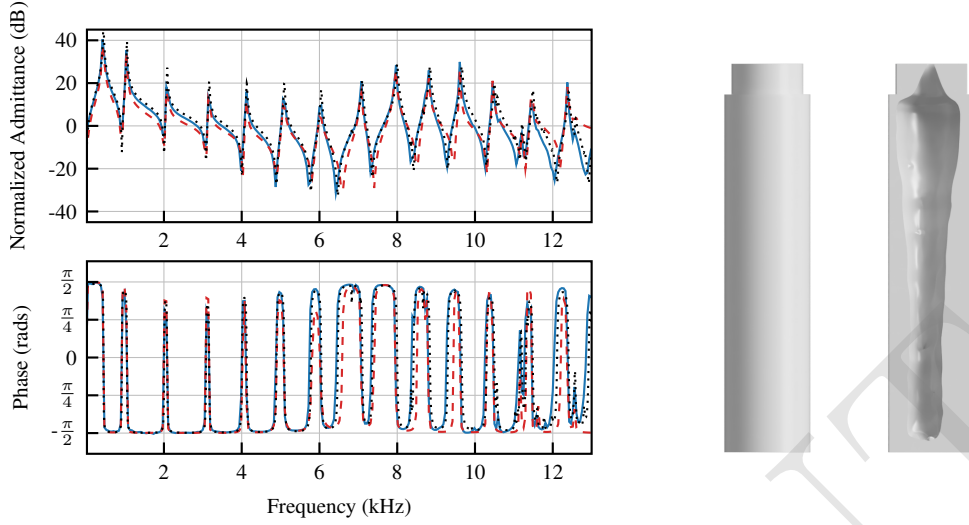


Figure 2: Left: Measurement of the instrument's input admittance in blue and corresponding modal model estimate in red, dashed. 3D FEM simulation in black, dotted. Right: 3D geometry used in the measurement and FEM simulation with cut away showing internal geometry.

4. CHARACTERIZATION OF THE PARACAS WHISTLE

To simulate the response of a real instrument using the simplified model presented in Section 2, it is necessary to estimate the modal bore admittance and mouth radiation impedance. The instrument was scanned in a Lumafield Neptune industrial CT scanner with a 120 kV X-ray source. Because the whistle is a stopped pipe, the excitation and radiation surfaces are shared. This makes it impossible to simultaneously measure the input admittance and radiation impedance. For this reason, the scan geometry was separated into an internal geometry to measure the input admittance and an external geometry to measure the mouth radiation impedance.

A. MODAL ADMITTANCE

To validate the measurements, a 3D FEM simulation of the geometry was carried out using a commercial FEM software package. Modal parameters were estimated using an unconstrained trust-region optimization. The modal fit was performed up to 13 kHz, and 20 modes were estimated. Figure 2 demonstrates the similar responses between the FEM simulations, measurements, and modal fit.

B. RADIATION IMPEDANCE

To measure the radiation impedance, the internal geometry was replaced with a simplified cylindrical geometry with radius 5.25 mm. The 3D scan of the instrument was sliced above its closed end so that the 3D printed geometry could be driven by the impedance probe from its base. This radiation impedance was also measured with a player's face near the radiating surface. The 3D printed geometry with and without a player is shown on the right of Figure 3.

After measuring the input impedance of the 3D-printed geometry, the radiation impedance was extracted using a transfer matrix of the internal cylindrical geometry. The mismatch between the theoretical and measured cylindrical impedance resulted in ripples in the extracted radiation impedance. These anomalies were smoothed out using Gaussian window averaging. Then, the theoretical radiation model was fit to the measured results. A 3D FEM simulation was also carried out up to 10 kHz to verify the measurement

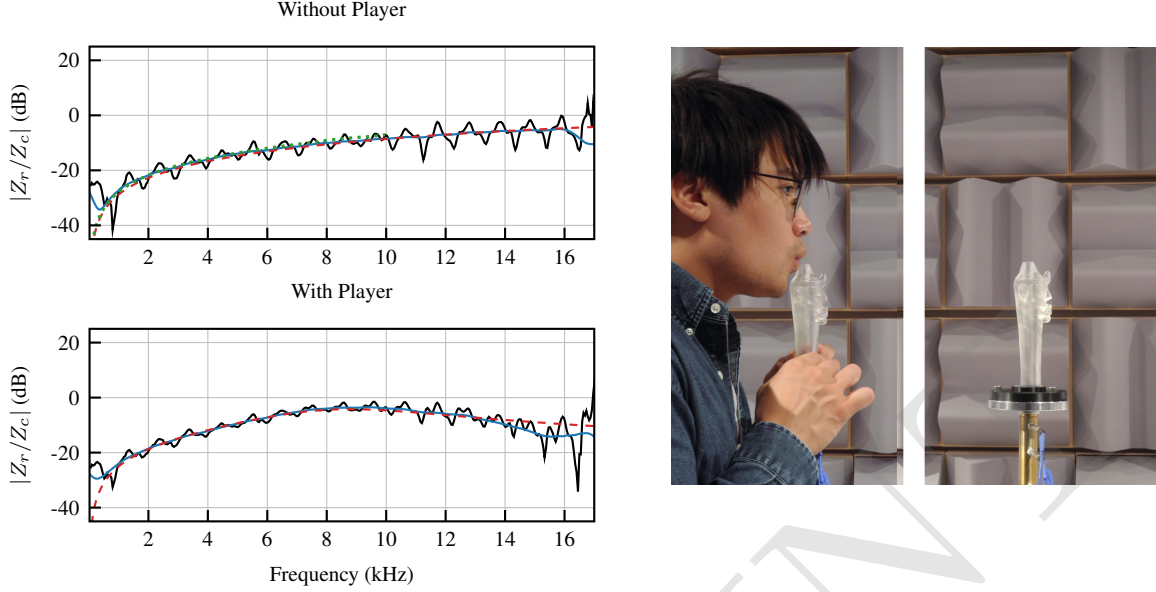


Figure 3: Left: The measured radiation impedance, in black, without (top) and with (bottom) the player. The blue curve is the smoothed response and the red dashed response is the model fit. Comparison with 3D FEM simulation without the player shown in dotted green. Right: The exterior geometry attached to the impedance probe with and without the player.

procedure. The measured and fitted radiation impedances are shown on the left of Figure 3. The radiation model and simulation agree well with the extracted radiation measurement without a player. A constrained optimization enforcing the realizability conditions in Eq. (16) was used to estimate the coefficients in Eq. (14) with the assumption $a_2 = b_2$. When the player is close to the instrument, this assumption is no longer valid as the radiation impedance does not approach the characteristic impedance, Z_c , in the high-frequency limit. Another constrained optimization was performed to estimate the radiation model coefficients without the assumption $a_2 = b_2$. The parameters for both models are given in Table 1.

5. RESULTS

Figure 4 displays two spectrograms comparing a recording made using a 3D printed replica of the whistle and a simulation of the mouth volume velocity, u_m , generated by the proposed physical model. The control jet velocity v_j used to generate the simulation is overlaid as a blue dashed line. The parameters used in the simulation were tuned heuristically. The mouth width is 1.1 mm and the distance to the labium is $2r_m$, where r_m is the radius of the open area equal to 5.25 mm. The simulation utilizes the radiation model that includes the player. The added noise a_t was set to 5^2 and the jet deflection coefficient, a_δ , was set to 0.05.

The model is capable of reproducing identical overblowing behavior when the jet velocity is increased. The tuning of the physical model nearly matches that of the recorded instrument. However, there are some

Table 1: Parameters extracted from the radiation impedance measurements.

	b_1	b_2	a_1	a_2
Without player	0.376	0.196	0.753	b_2
With player	0.575 ⁹	0.0554	0.937	1.387

notable differences between the recorded and simulated signals. The spectral content of the simulated signal is much stronger compared to the recording. This may be because the simulation is generating the volume flow at the mouth instead of the pressure at a microphone at a distance away from the mouth. A more significant difference is the character of the noise in the simulation. While including the dependence on the average cross-flow velocity, $\langle v_m \rangle$, aids in increasing the amount of noise during overblowing, the simulated signal lacks the diffuse noise in the recorded signal. The spectral content of the added synthetic noise also differs from the recorded signal.

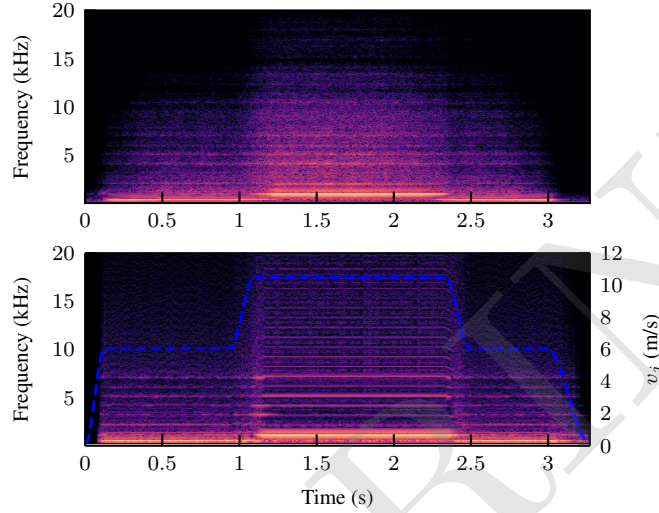


Figure 4: *Top: Spectrogram of a recording of the 3D printed instrument. Bottom: Re-synthesis with v_j profile used to generate re-synthesis shown in blue, dashed.*

6. CONCLUSION & FUTURE WORK

We have presented a simplified model of a stopped pipe excited by a jet. The model is described using the pH system framework and discretized with FDTD methods. Added noise and deflection of the jet due to crossflow are computed using a running RMS level detector. The body admittance is described using a modal representation and the radiation impedance is realized using a lumped model. The model is capable of similar overblowing behavior to the instrument being modeled. The proposed model could be adapted for simulations of laminar jets in other flute and recorder-like instruments by removing aspects specific to stopped pipes. The noise model is lacking and remains for future work. An interesting avenue of exploration would be to incorporate a vortex sound simulation into the model.¹⁶ A signal-based model based off of playing measurements would be a more ready solution. The algorithm proposed here is lightweight and is capable of real-time simulations; an interactive version of the model is left to future work.

By utilizing a CT scanner, the internal and external geometry of the instrument being studied could be separated. This enabled measurements of an internal body admittance and external radiation impedance with a player's face next to the instrument. The same CT scanning, 3D printing, and measurement pipeline presented here has applications to the characterization of other fragile historical instruments. Even in the case of instruments that can be measured directly, CT scanning can be useful for isolating different parts of an instrument's geometry and studying their individual contribution to an instrument's sound.

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